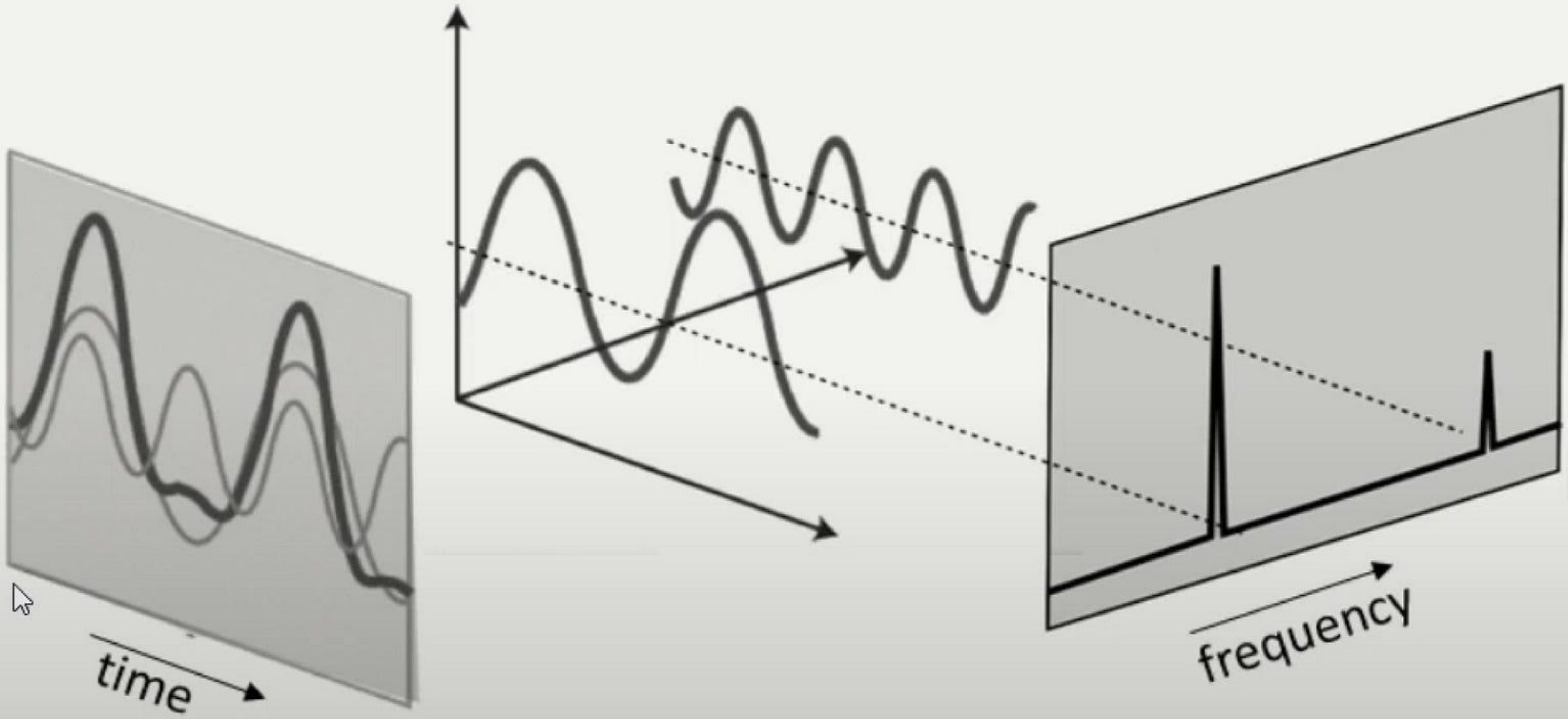
Frequency Domain Image Filtering

By

Dr. Abdul Rehman Abbasi

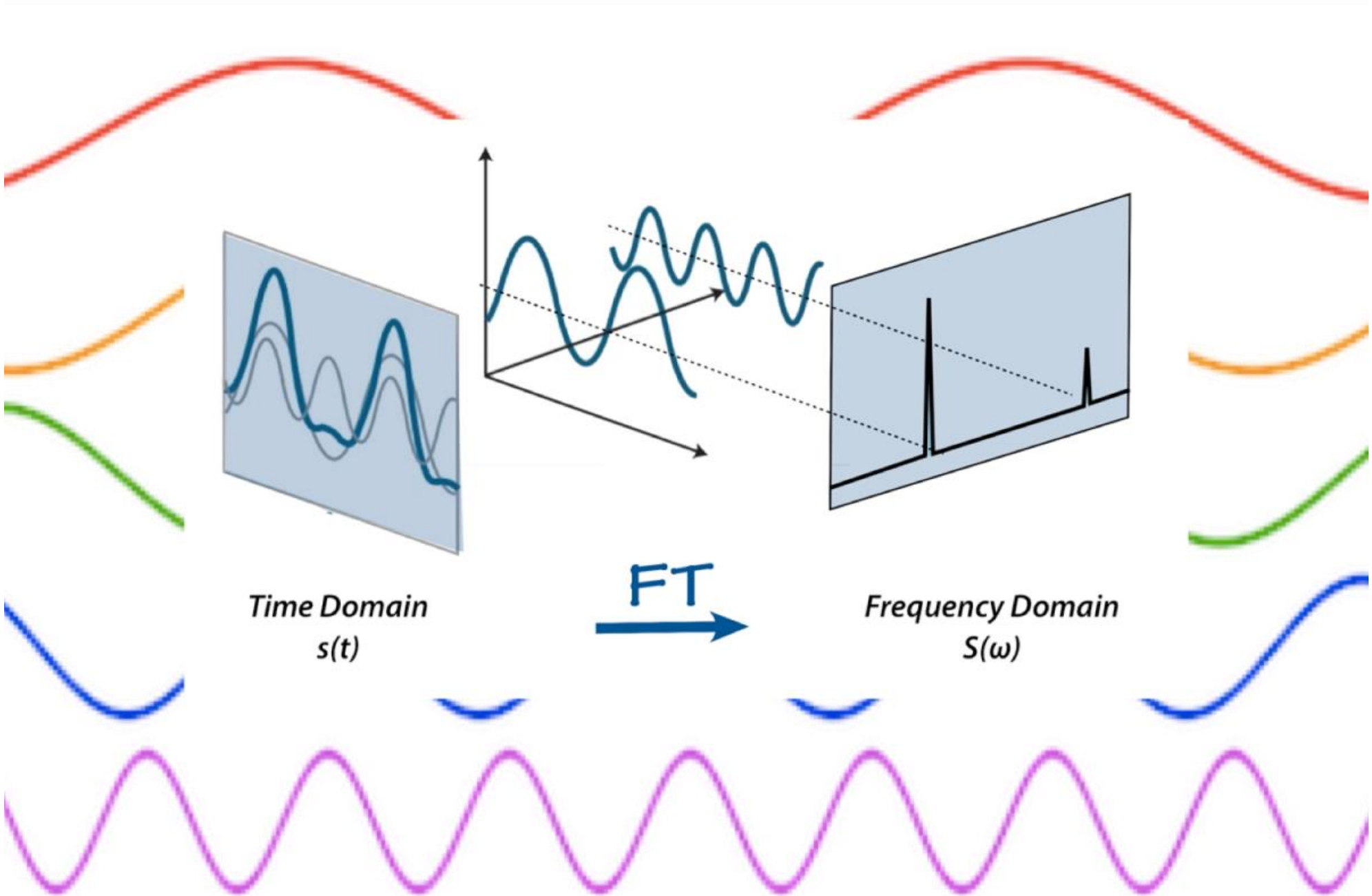
Time & Frequency Domain Operations

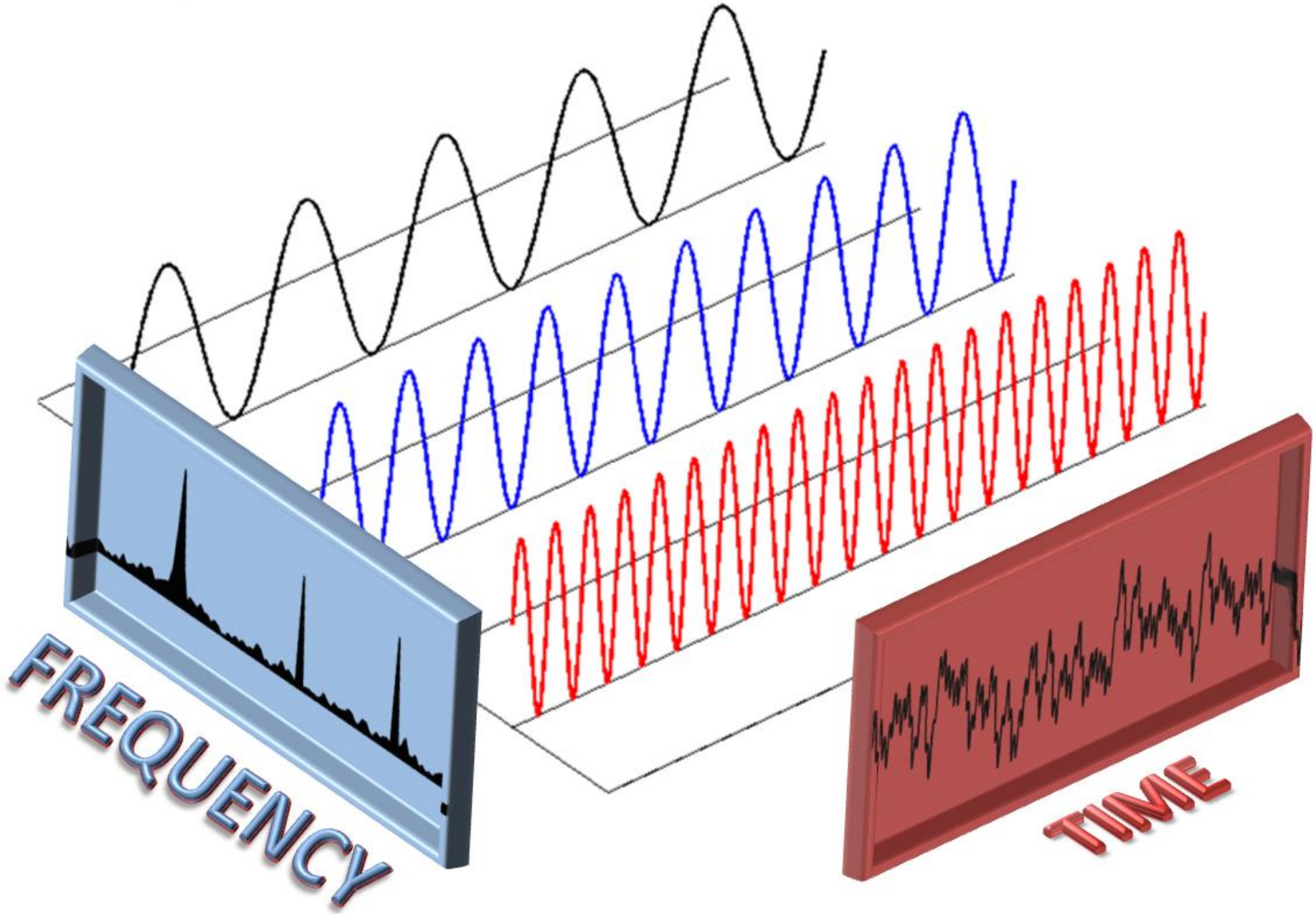


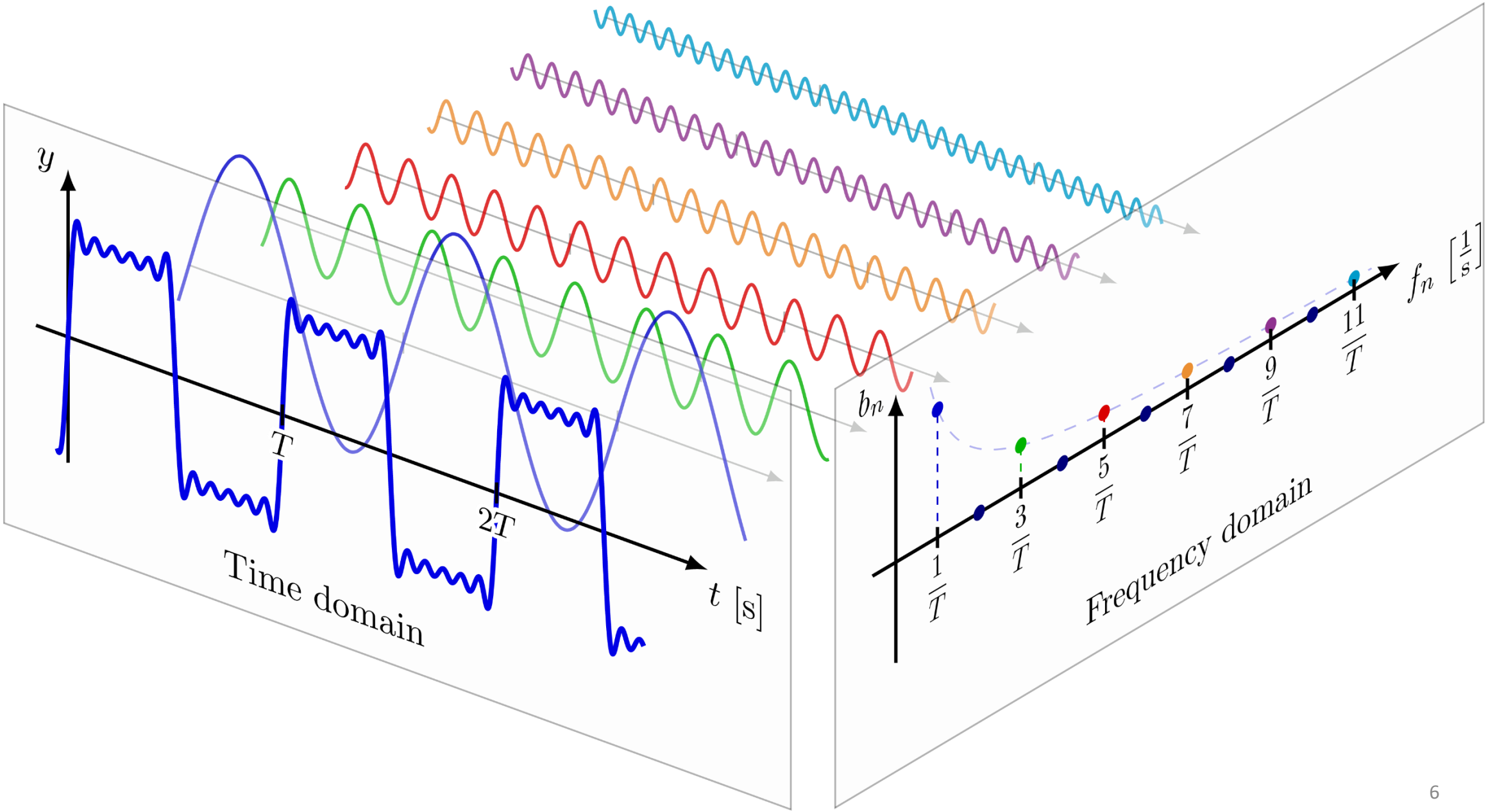
Mixture of individual sine wave with different frequencies

Separated individual sine waves

Frequencies for respective sine waves


FT
Time Domain
s(t)
Frequency Domain
S(ω)


FREQUENCY
TIME


y
T
2T
t [s]
Time domain
b_n
$\frac{1}{T}$
$\frac{3}{T}$
$\frac{5}{T}$
$\frac{7}{T}$
$\frac{9}{T}$
$\frac{11}{T}$
$f_n \left[\frac{1}{s}\right]$
Frequency domain

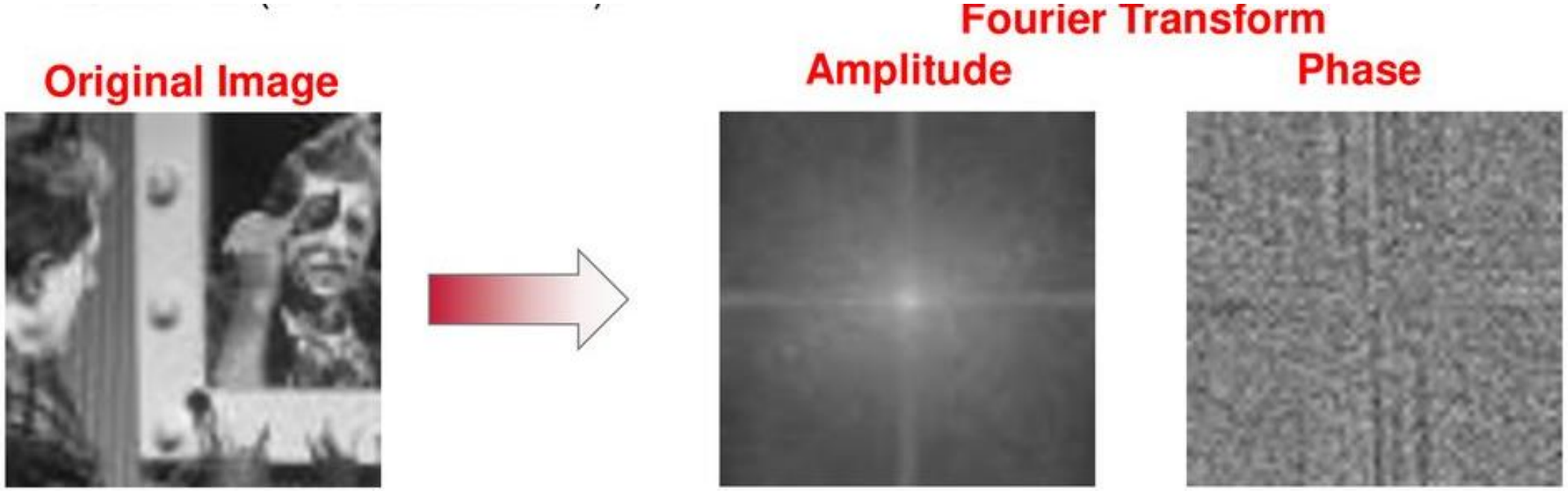
Time & Frequency Domain Operations

The *Discrete Fourier Transform* of $f(x, y)$, for x = 0, 1, 2…M-1 and y = 0,1,2…N-1, **denoted** by $F(u, v)$, is given by the equation:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

for u = 0, 1, 2…M-1 and v = 0, 1, 2…N-1.

Time & Frequency Domain Operations



1. What are the key differences between spatial domain and frequency domain filtering?

Spatial domain filtering operates directly on image pixels using convolution masks, while frequency domain filtering modifies the image's frequency components via the Fourier Transform.

Spatial domain filtering involves modifying the pixel values of an image based on the values of neighboring pixels using techniques like convolution. Common spatial filters include:

Smoothing filters (e.g., averaging, Gaussian) to reduce noise

Sharpening filters (e.g., Laplacian, Sobel) to enhance edges

What are the key differences between spatial domain and frequency domain filtering?

Frequency domain filtering, on the other hand, first transforms the image into the frequency domain using the **Fourier Transform**.

The image is represented by its sinusoidal frequency components.

Filtering is done by modifying these frequency components (typically by multiplying them with a filter), and the result is transformed back to the spatial domain using the **Inverse Fourier Transform**.

What are the key differences between spatial domain and frequency domain filtering?

Suppose you have a grayscale image of a printed document that contains both text and a patterned background.

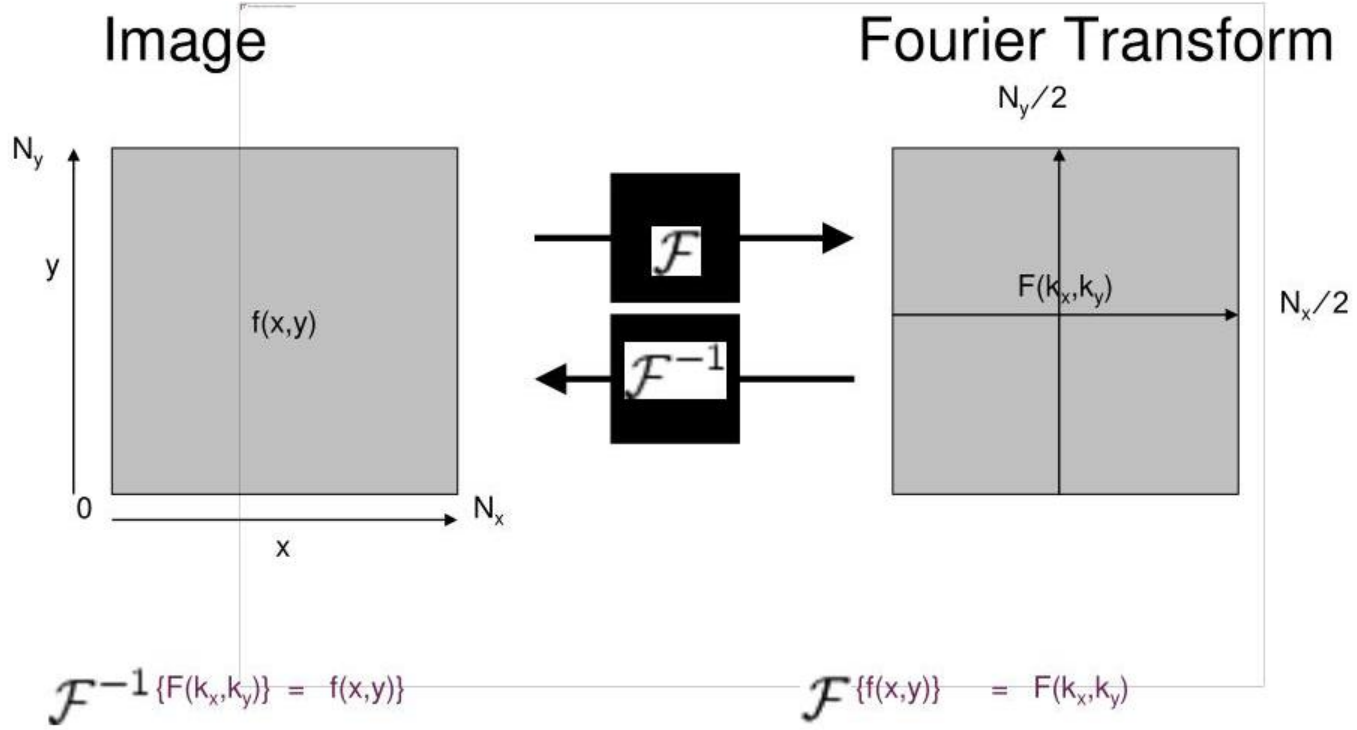
- If you apply a **low-pass spatial filter**, it will blur the whole image, reducing both the text sharpness and the background noise.
- If you apply a **frequency domain high-pass filter**, you can specifically suppress low-frequency background patterns while preserving or even enhancing the high-frequency text details.

What are the key differences between spatial domain and frequency domain filtering?

Filtering Method	Effect on Image
Spatial Gaussian Low-Pass Filter	Smooths image, reduces noise, blurs edges
Frequency Gaussian Low-Pass Filter	Smooths image, reduces noise, blurs edges
Frequency Gaussian High-Pass Filter	Enhances edges and fine details

2. How does the Fourier Transform help in separating image components based on frequency?

- The Fourier Transform decomposes an image into sinusoidal components, revealing which spatial frequencies are present and allowing for separation based on those frequencies.

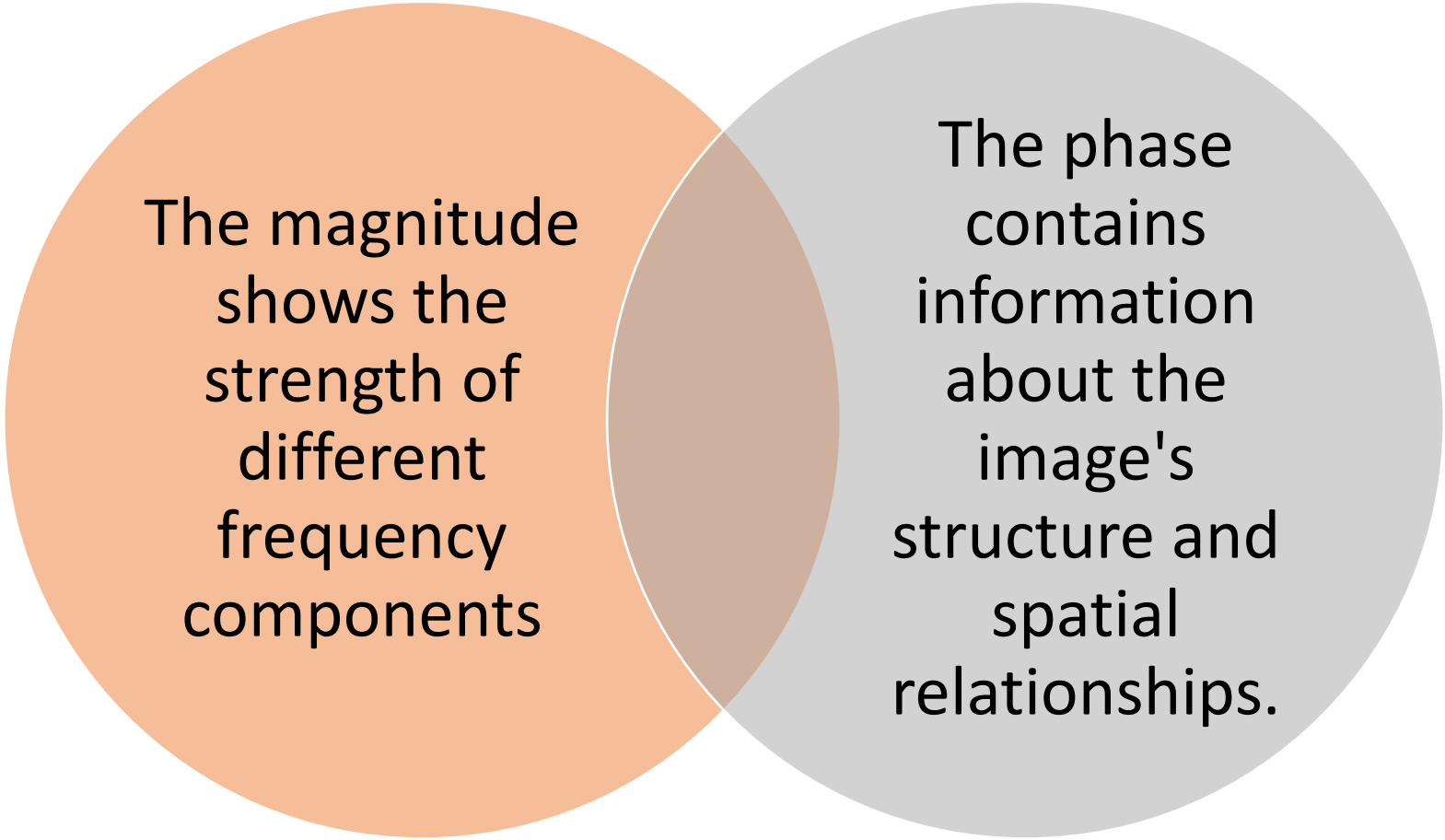


2. How does the Fourier Transform help in separating image components based on frequency?

Figure 1 displays an image and its corresponding Fourier transform.
What is the problem with this image? How we can solve it?

Figure 1: Original image and its Fourier transform

3. What is the significance of the magnitude and phase components in the Fourier Transform?


The magnitude shows the strength of different frequency components
The phase contains information about the image's structure and spatial relationships.

4. What types of filters are commonly used in the frequency domain, and what are their respective purposes?

Common filters include Low-Pass (blurring)

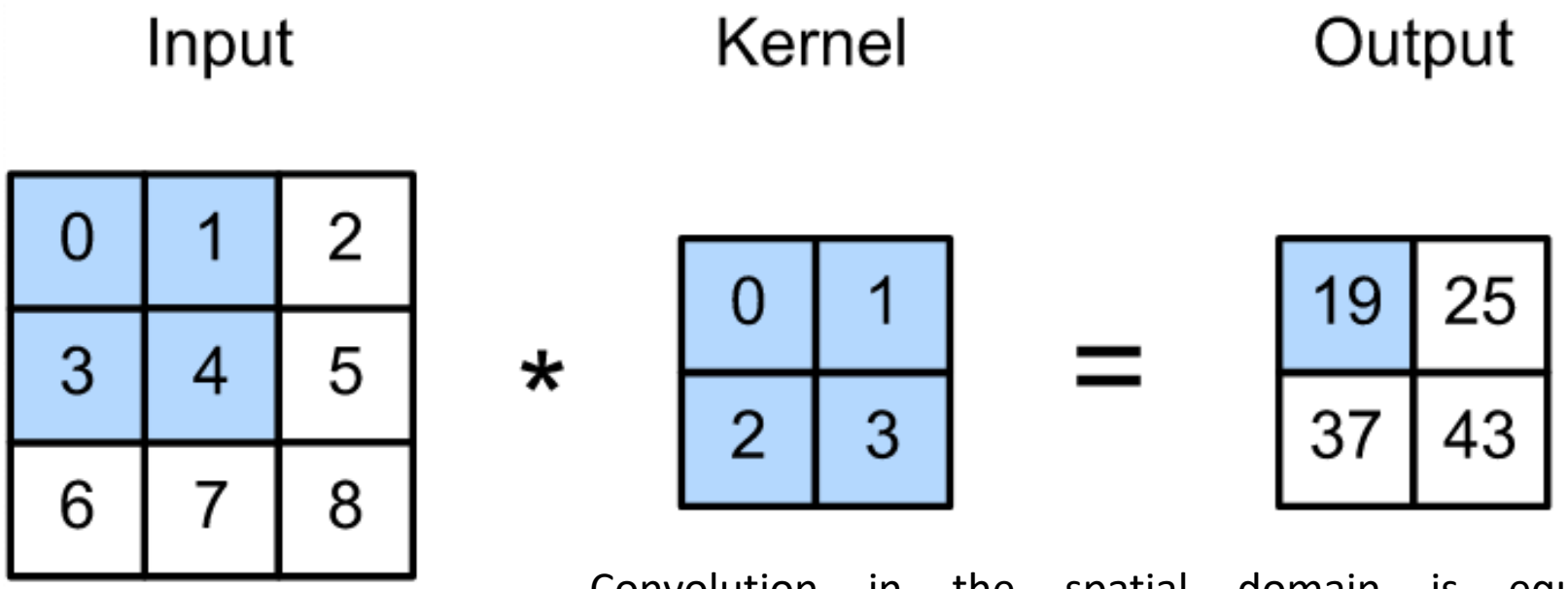
High-Pass (edge detection)

Band-Pass (feature isolation)

Band-Stop (noise removal)

5. Why is frequency domain filtering often preferred for certain types of image processing tasks?

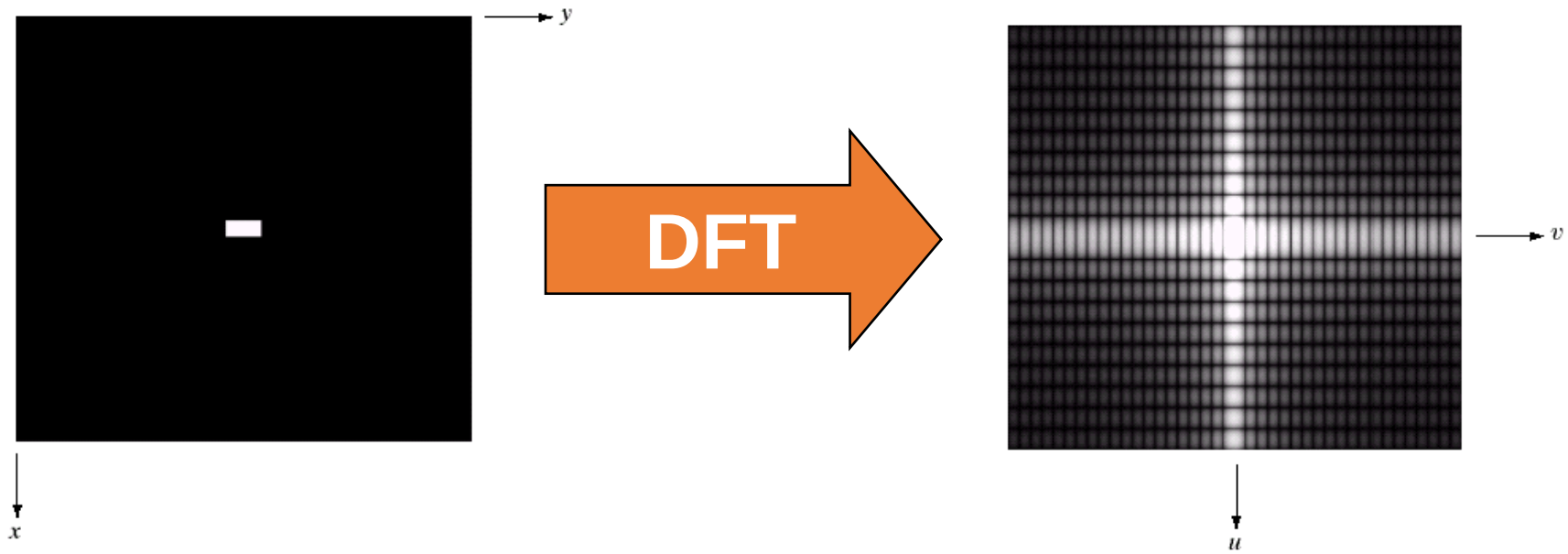
- It simplifies operations like convolution into multiplications, making it computationally efficient and effective for large kernel sizes.



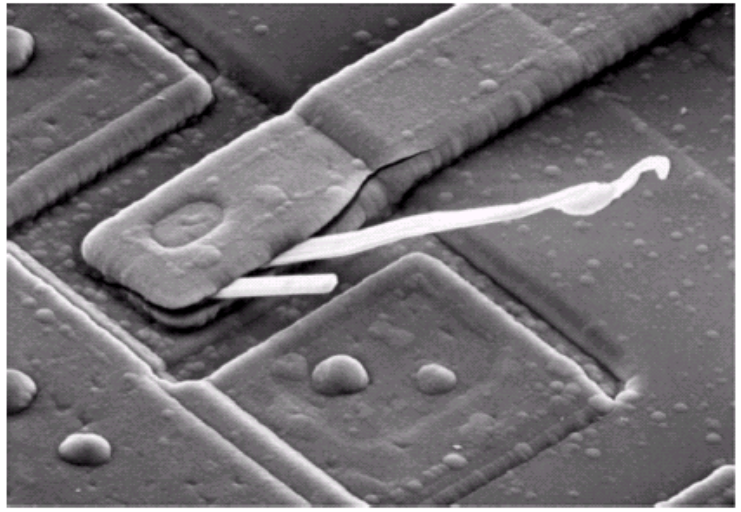
Convolution in the spatial domain is equivalent to multiplication in the frequency domain, making certain processes more efficient.

Discrete Fourier Transform

The DFT of a two dimensional image can be visualised by showing the spectrum of the images component frequencies

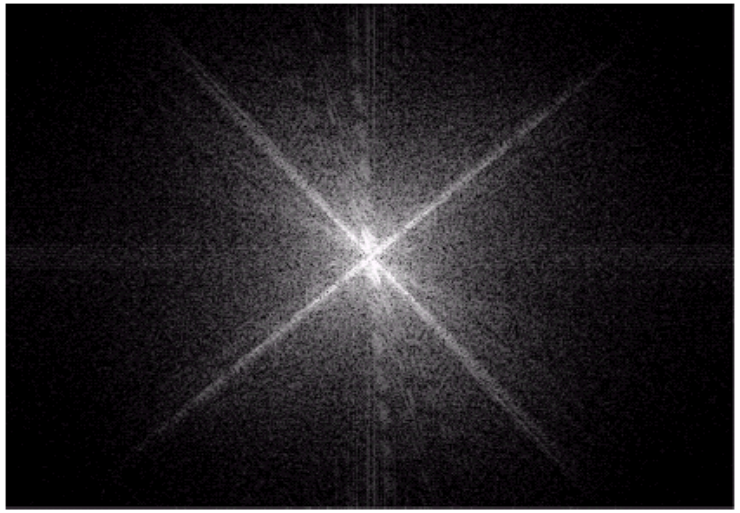


Discrete Fourier Transform



Scanning electron microscope image of an integrated circuit magnified ~2500 times

DFT



Fourier spectrum of the image

Inverse DFT

It is really important to note that the Fourier transform is completely **reversible**

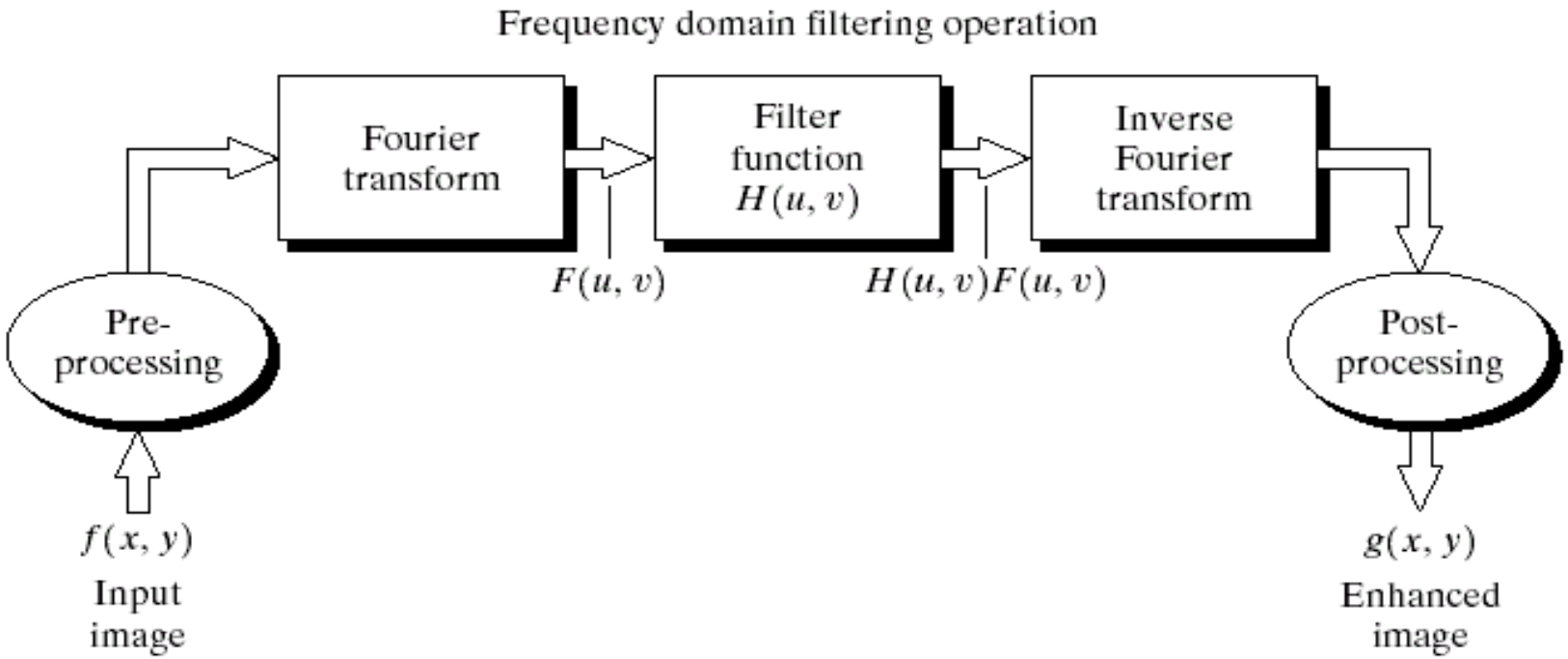
The inverse DFT is given by:

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M + vy/N)}$$

for x = 0, 1, 2…M-1 and y = 0, 1, 2…N-1

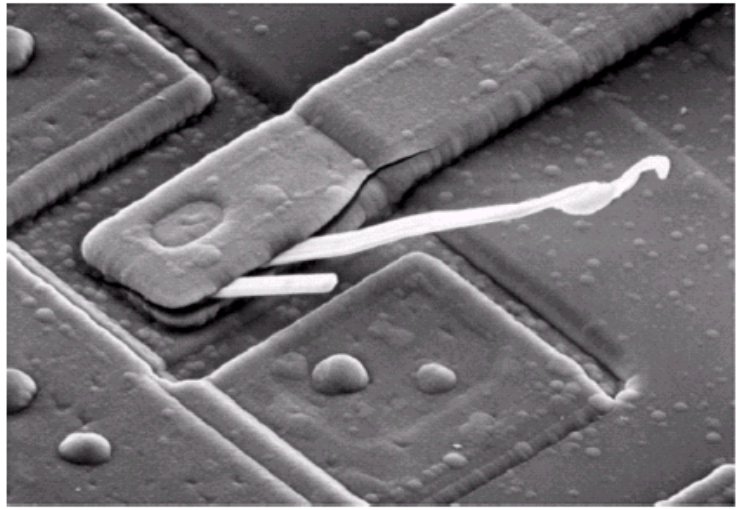
To filter an image in the frequency domain:

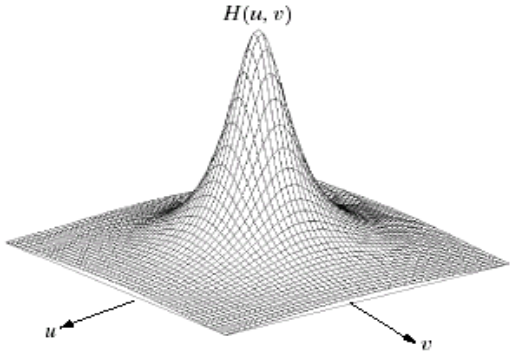
1. Compute $F(u,v)$ the DFT of the image
2. Multiply $F(u,v)$ by a filter function $H(u,v)$
3. Compute the inverse DFT of the result

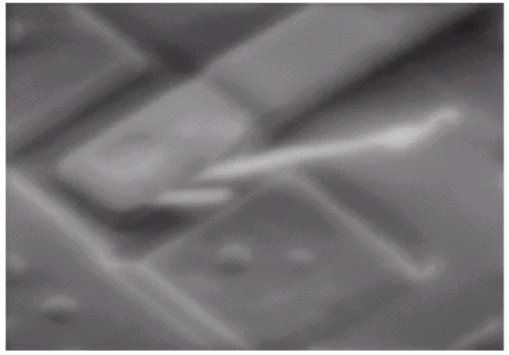


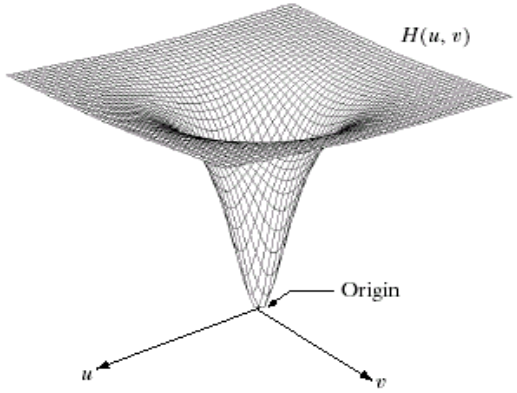
Frequency Domain Filters

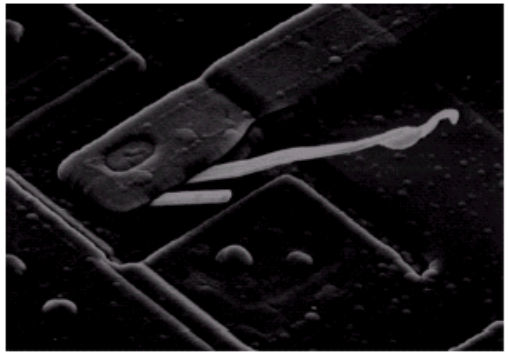
Low Pass Filter











High Pass Filter

Smoothing Frequency Domain Filters

Smoothing is achieved in the frequency domain by dropping out the high frequency components

The basic model for filtering is:

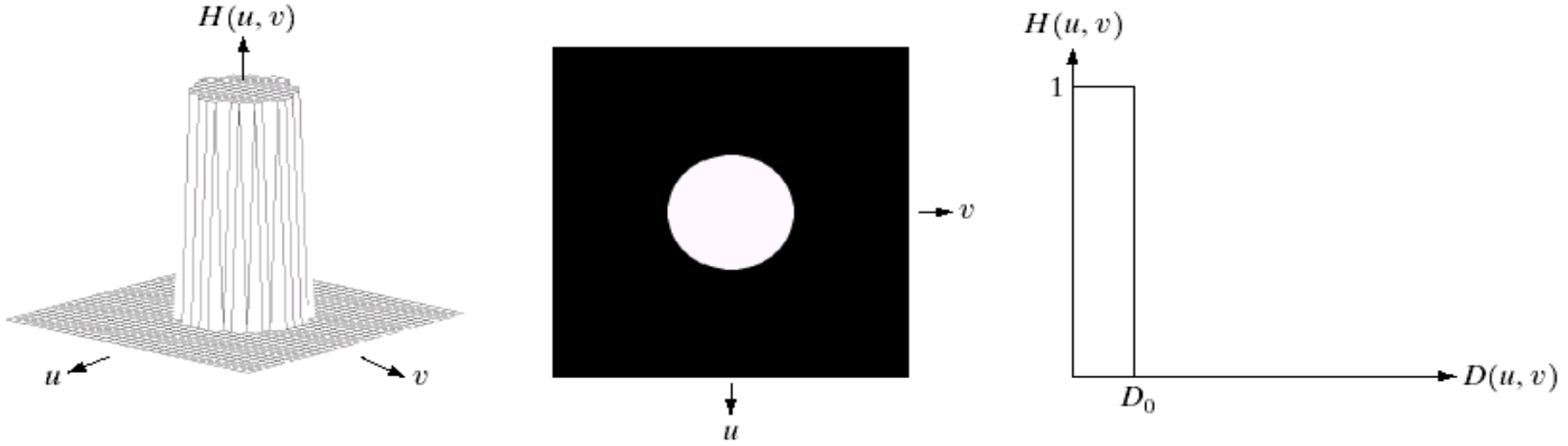
$$G(u,v) = H(u,v)F(u,v)$$

where $F(u,v)$ is the Fourier transform of the image being filtered and $H(u,v)$ is the filter transform function

Low pass filters – only pass the low frequencies, drop the high ones

Ideal Low Pass Filter

Simply cut off all high frequency components that are a specified distance D_0 from the origin of the transform



Ideal Low Pass Filter

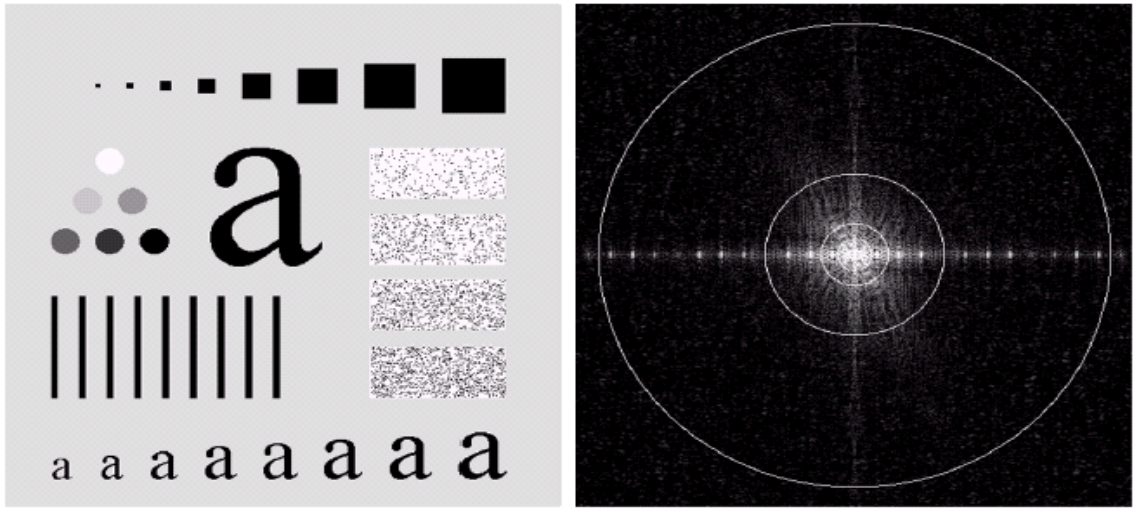
The transfer function for the ideal low pass filter can be given as:

$$H(u,v) = \begin{cases} 1 \text{ if } D(u,v) \le D_0 \\ 0 \text{ if } D(u,v) > D_0 \end{cases}$$

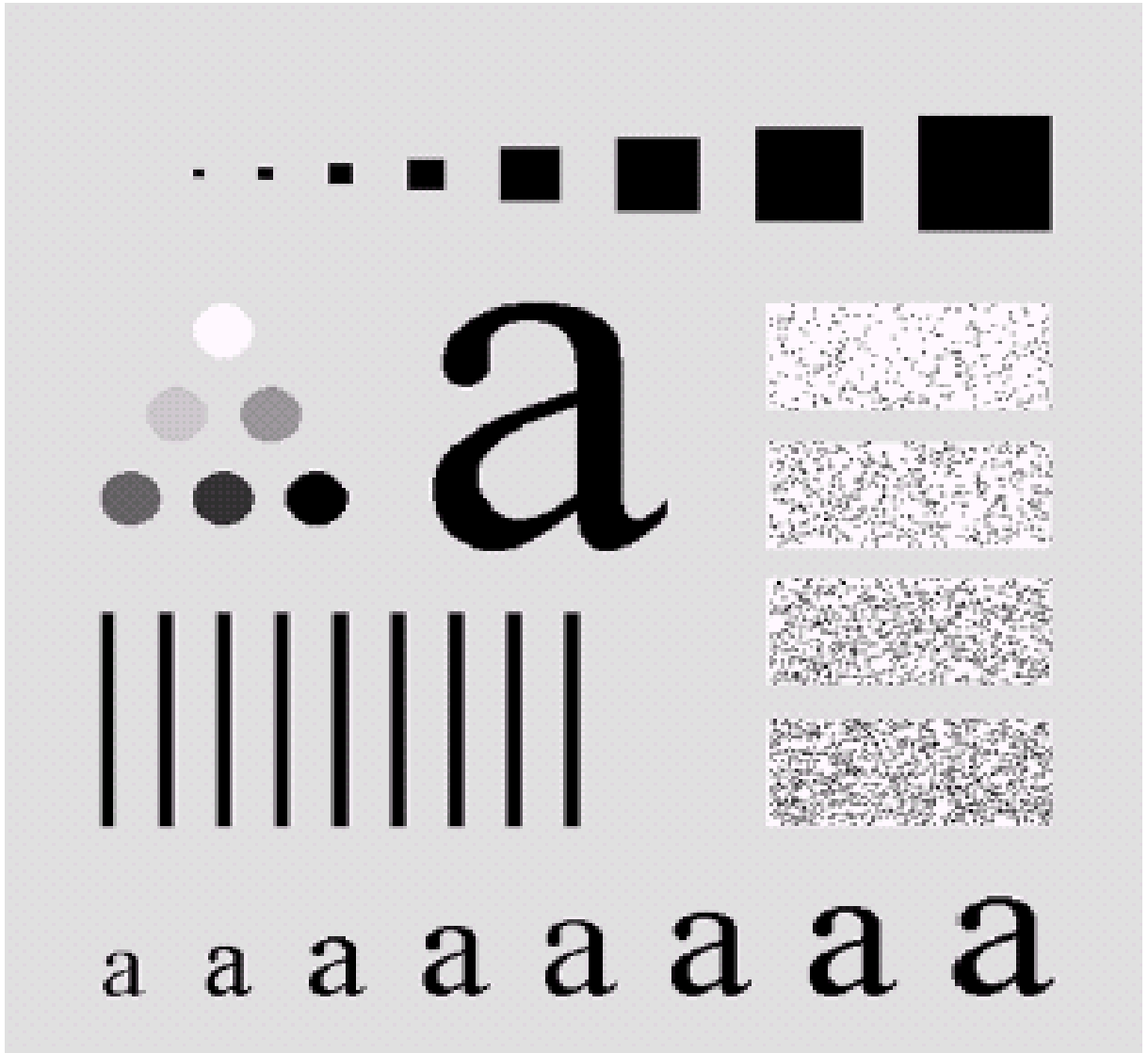
where $D(u,v)$ is given as:

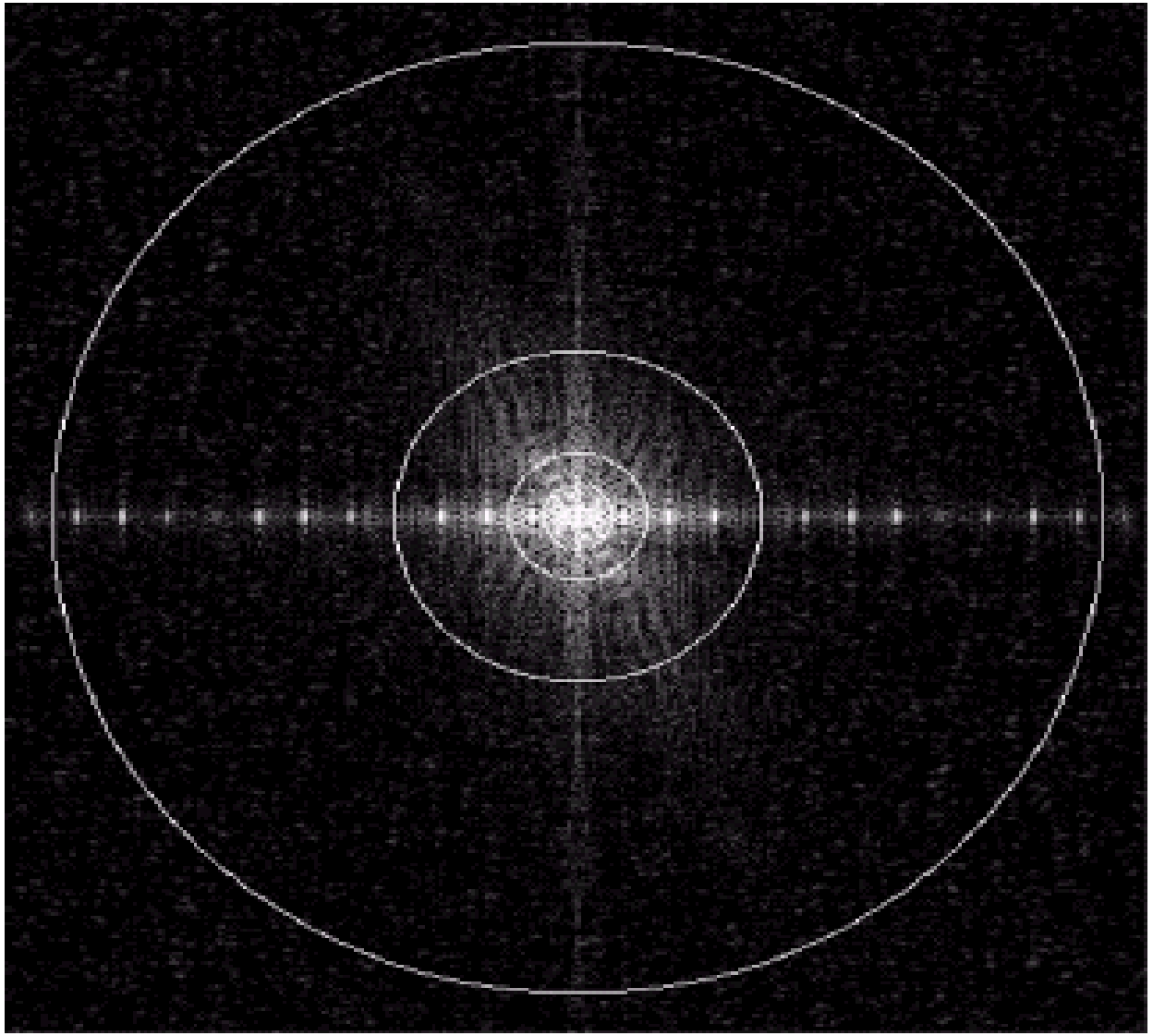
$$D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$$

Ideal Low Pass Filter

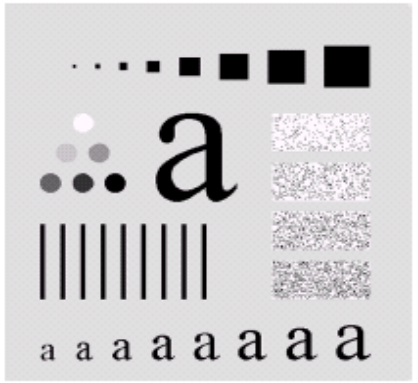


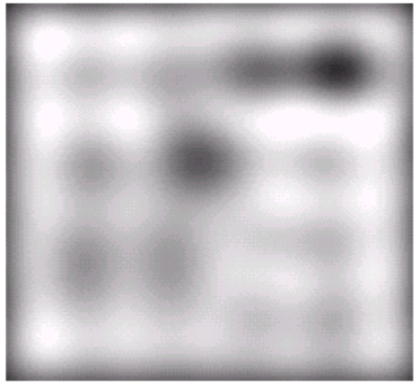
Above we show an image, it’s Fourier spectrum and a series of ideal low pass filters of radius 5, 15, 30, 80 and 230 superimposed on top of it





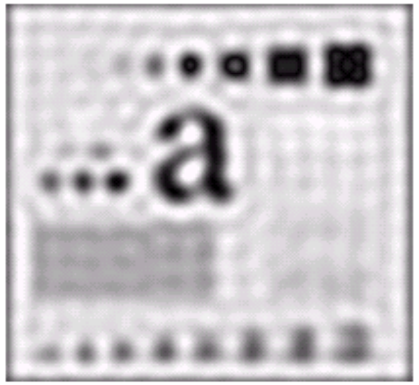
Original image

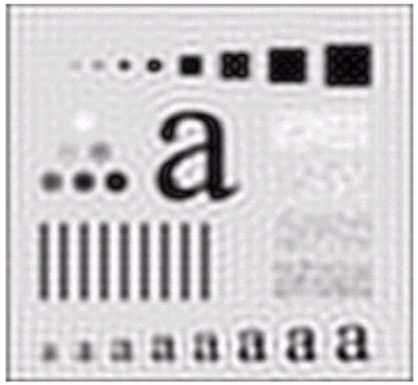




Result of filtering with ideal low pass filter of radius 5

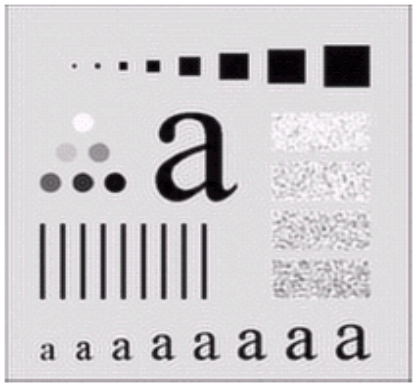
Result of filtering with ideal low pass filter of radius 15

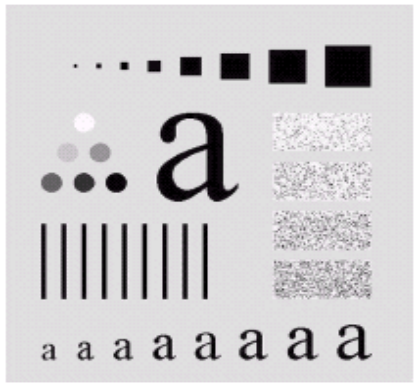




Result of filtering with ideal low pass filter of radius 30

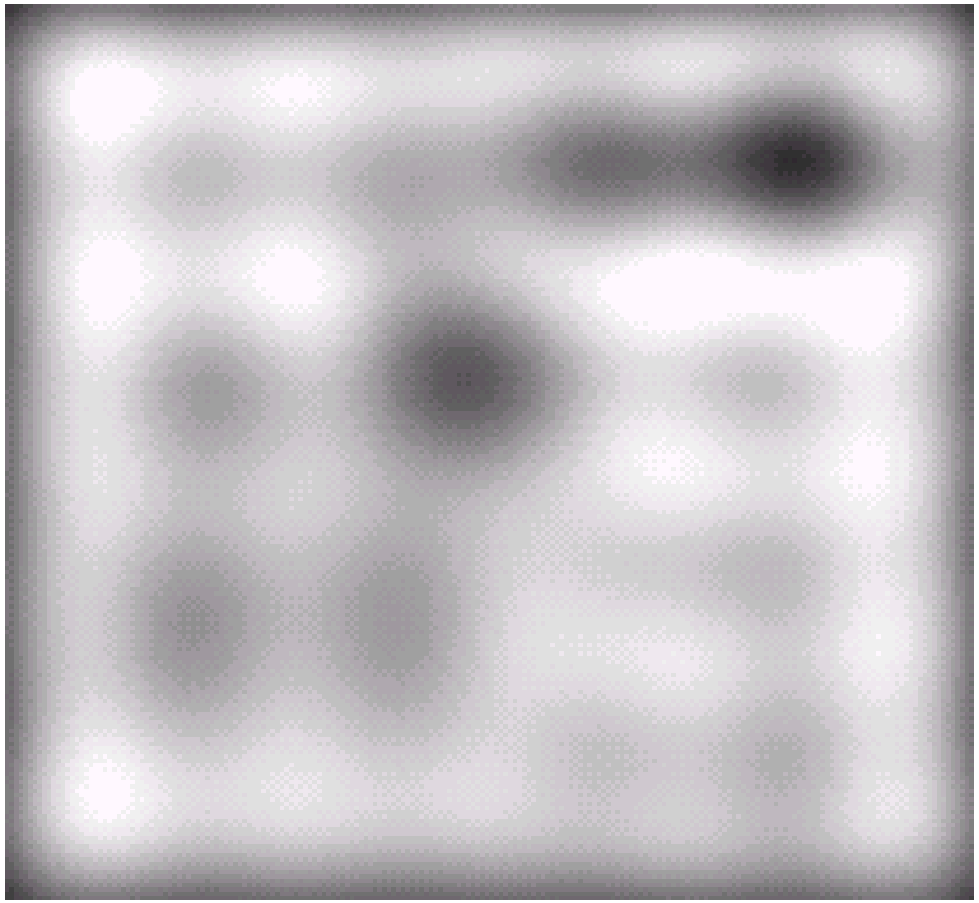
Result of filtering with ideal low pass filter of radius 80



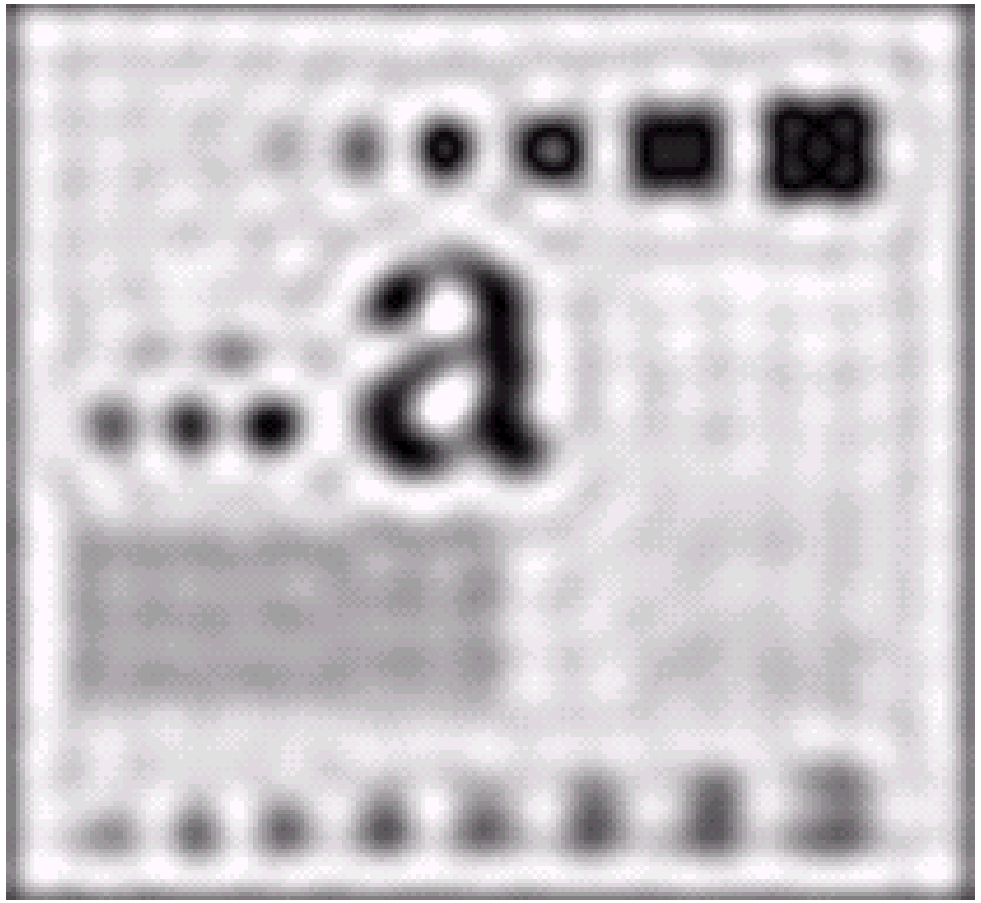


Result of filtering with ideal low pass filter of radius 230

Ideal Low Pass Filter (cont…)



Result of filtering with ideal low pass filter of radius 5

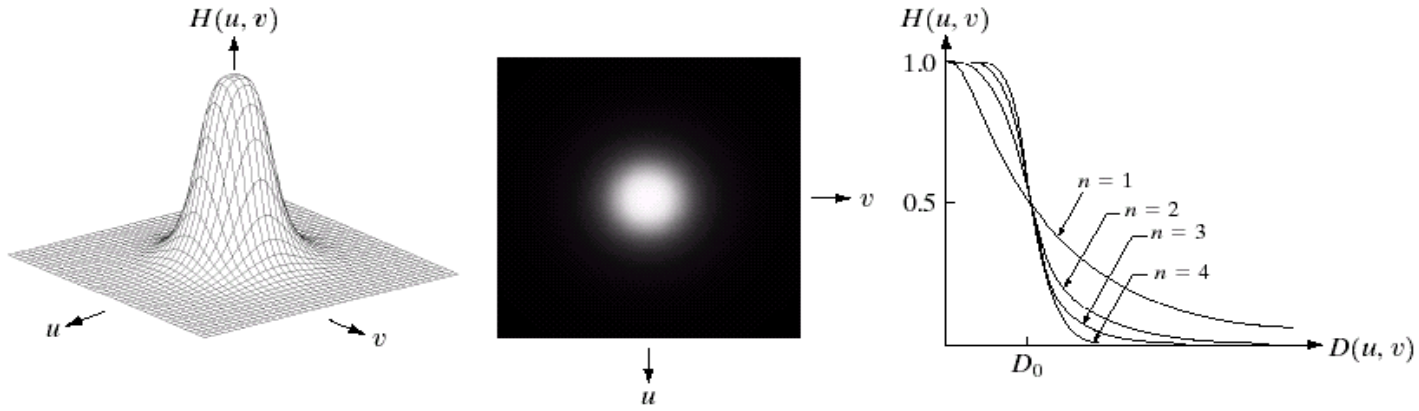


Result of filtering with ideal low pass filter of radius 15

Butterworth Lowpass Filters

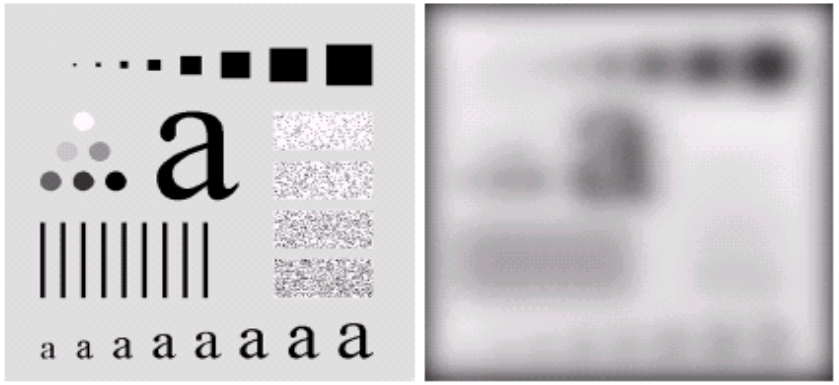
The transfer function of a Butterworth lowpass filter of order n with cutoff frequency at distance D_0 from the origin is defined as:

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$



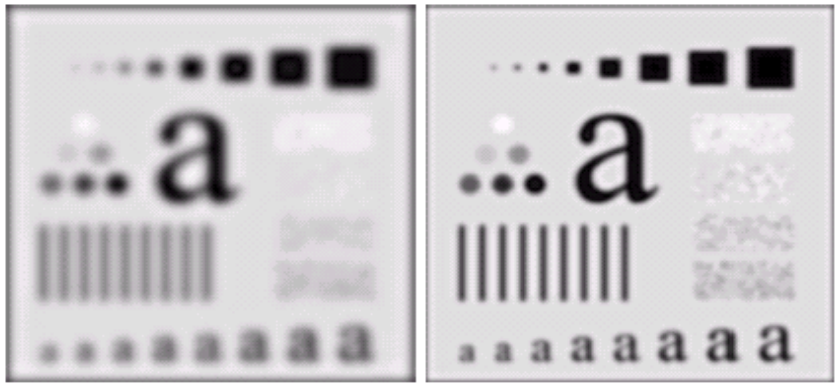
Butterworth Low Pass Filter

Original image

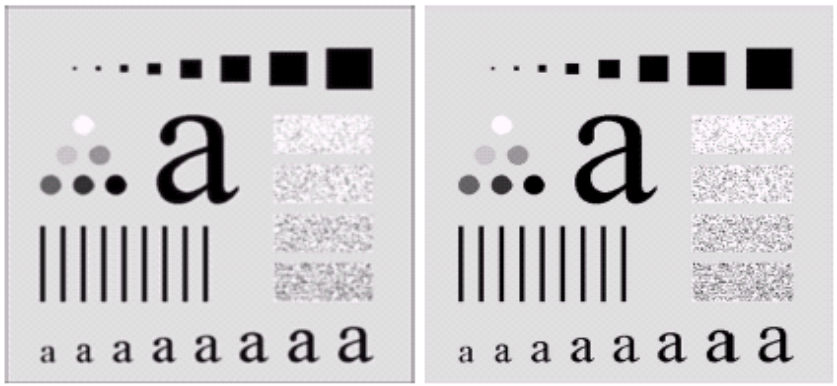


Result of filtering with Butterworth filter of order 2 and cutoff radius 5

Result of filtering with Butterworth filter of order 2 and cutoff radius 15

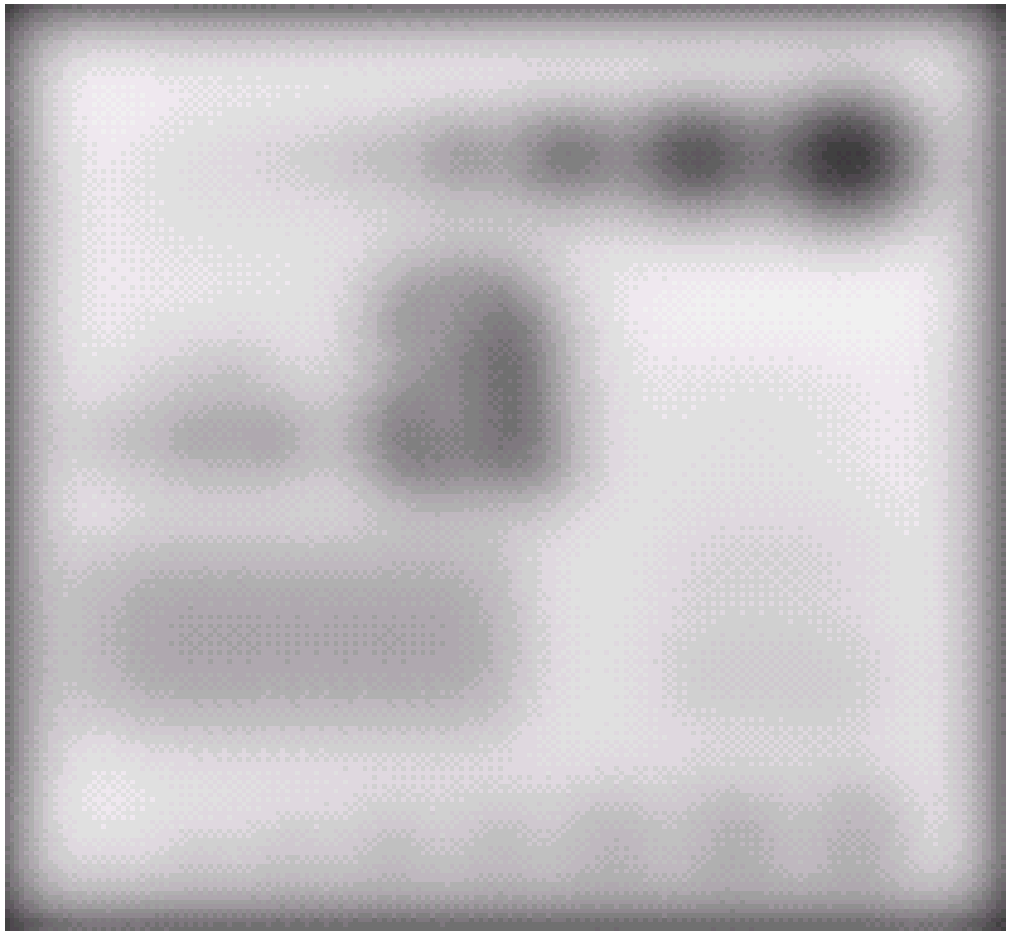


Result of filtering with Butterworth filter of order 2 and cutoff radius 80



Result of filtering with Butterworth filter of order 2 and cutoff radius 230

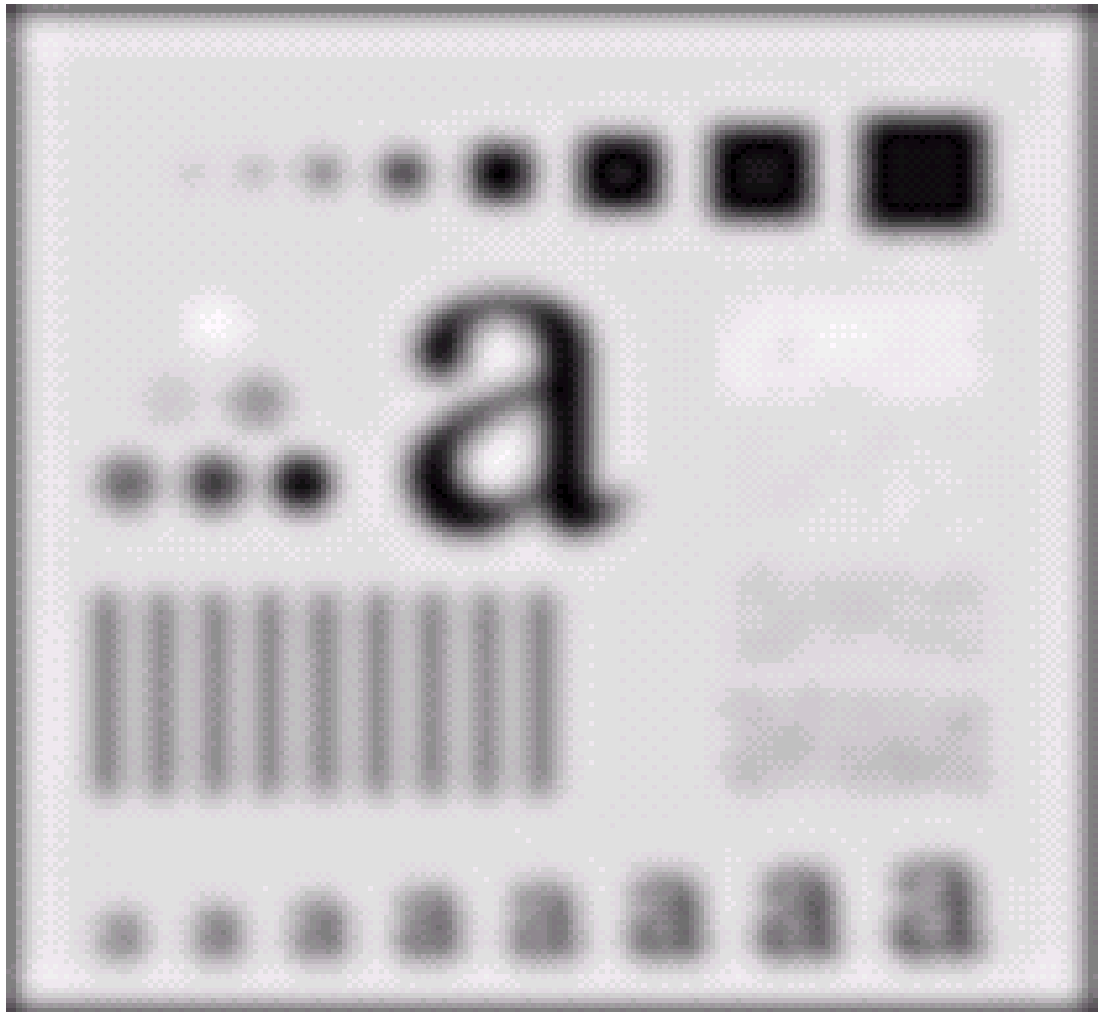
Butterworth Low Pass Filter



Result of filtering with Butterworth filter of order 2 and cutoff radius 5

Butterworth Low Pass Filter

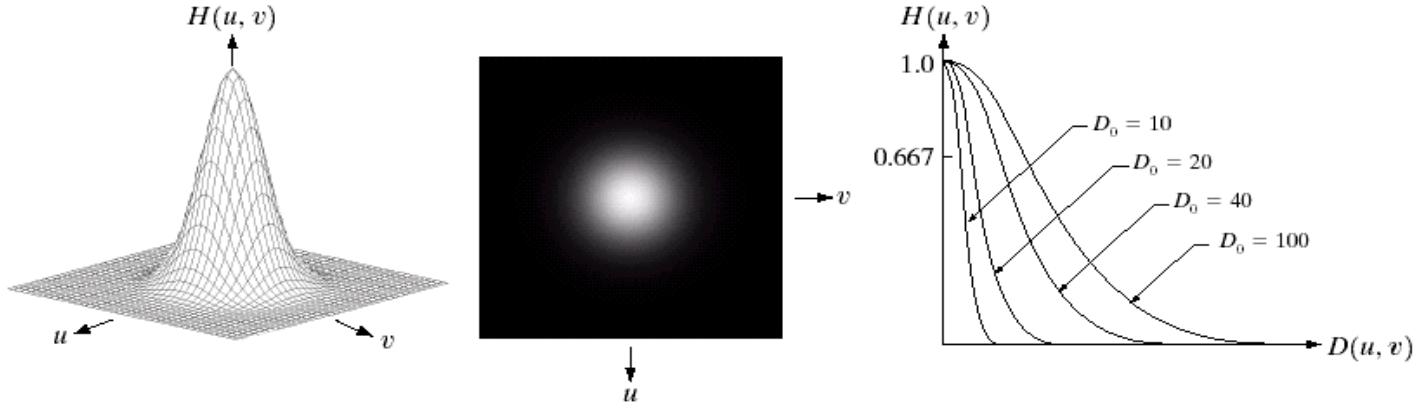
Result of filtering with Butterworth filter of order 2 and cutoff radius 15



Gaussian Lowpass Filter

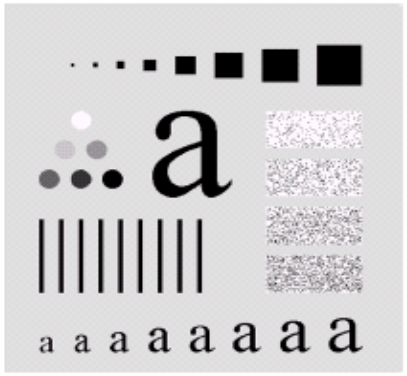
The transfer function of a Gaussian lowpass filter is defined as:

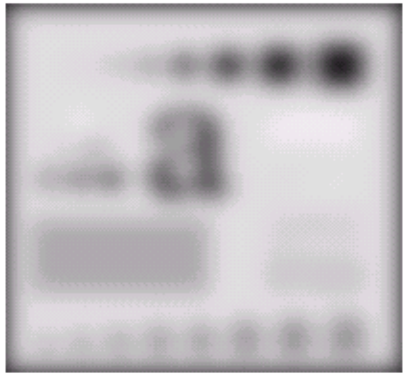
$$H(u,v) = e^{-D^2(u,v)/2D_0{}^2}$$



Gaussian Lowpass Filter

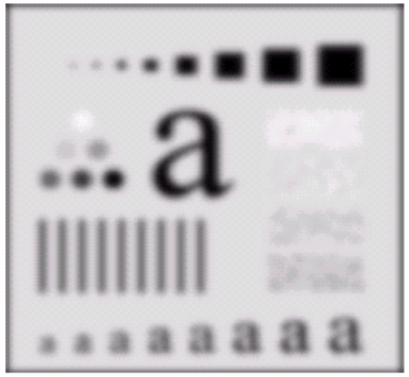
Original image

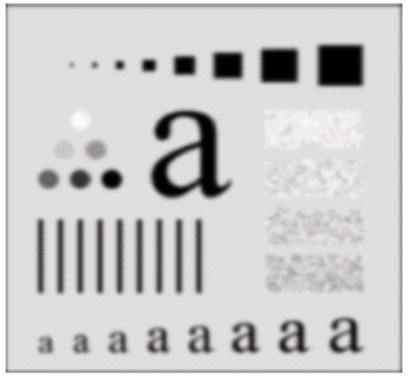




Result of filtering with Gaussian filter with cutoff radius 5

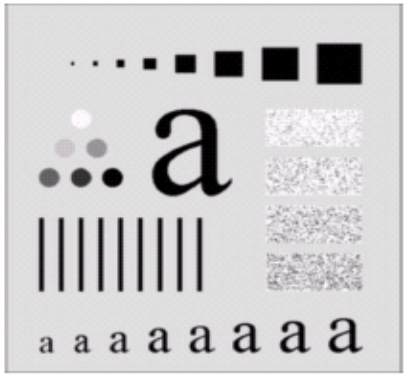
Result of filtering with Gaussian filter with cutoff radius 15

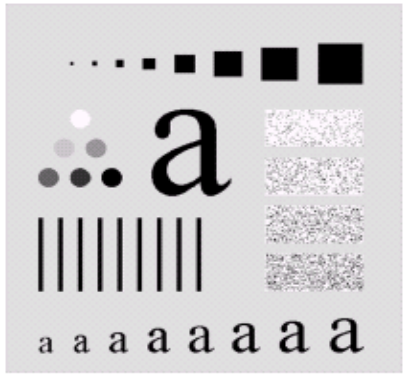




Result of filtering with Gaussian filter with cutoff radius 30

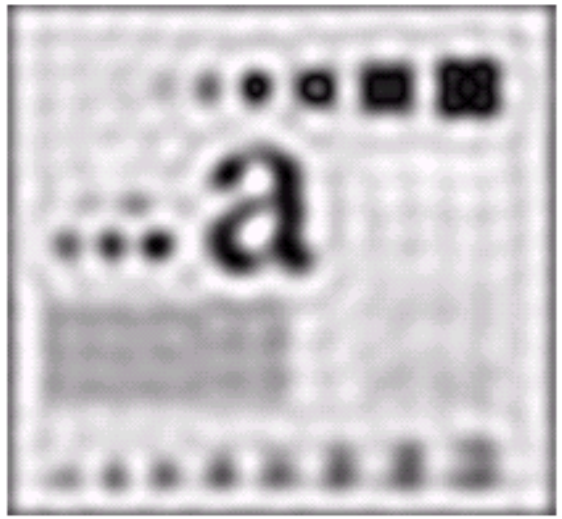
Result of filtering with Gaussian filter with cutoff radius 85

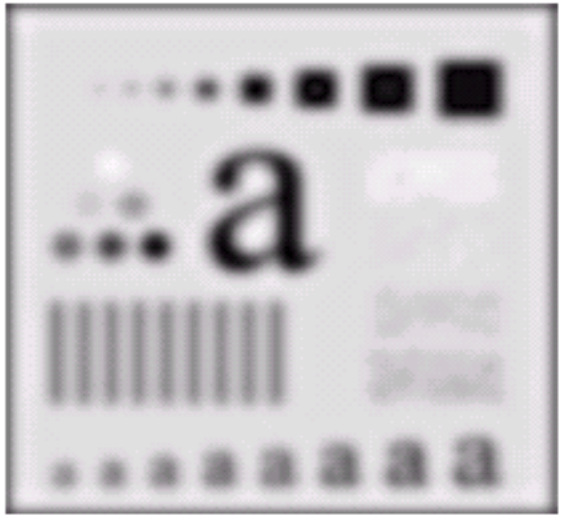




Result of filtering with Gaussian filter with cutoff radius 230

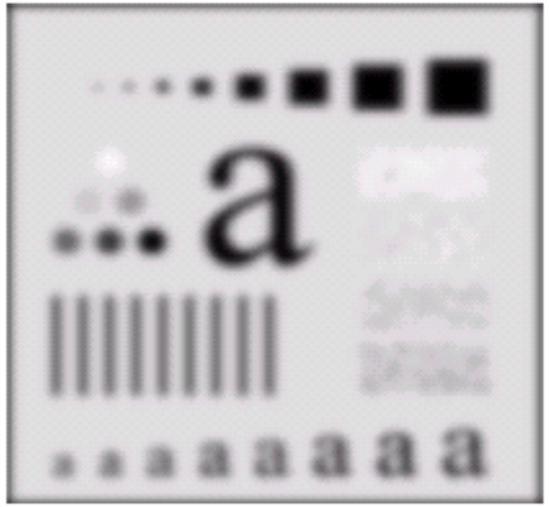
Result of filtering with ideal low pass filter of radius 15





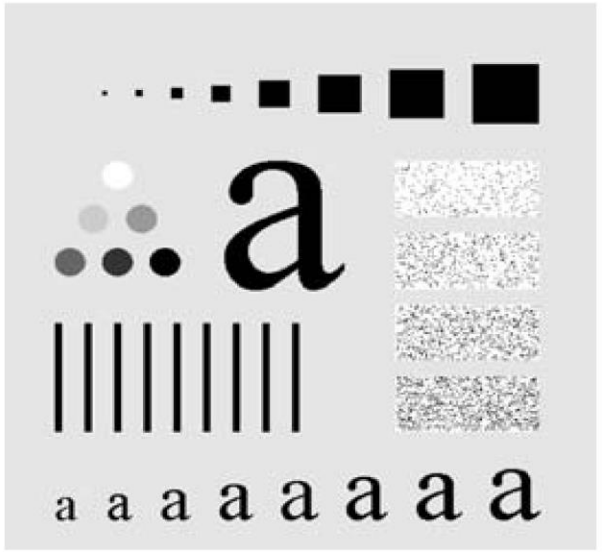
Result of filtering with Butterworth filter of order 2 and cutoff radius 15

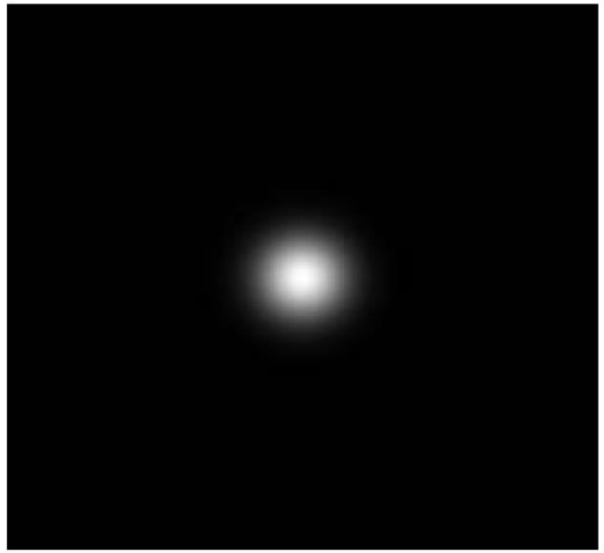
Result of filtering with Gaussian filter with cutoff radius 15



Lowpass Filtering Examples (cont…)

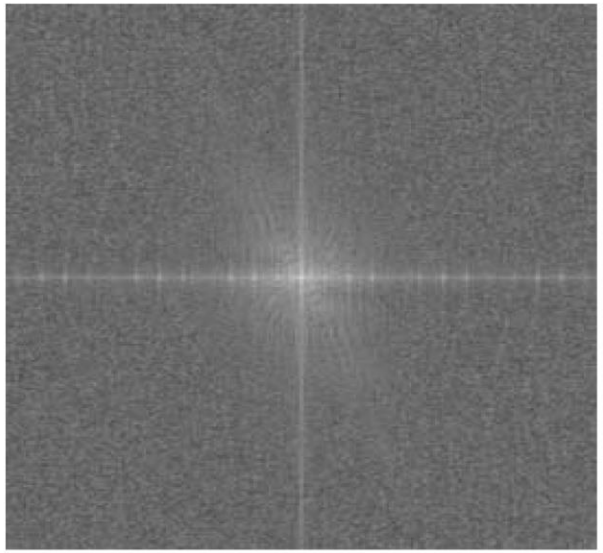
Original image





Gaussian lowpass filter

Spectrum of original image



Processed image

Sharpening in the Frequency Domain

Edges and fine detail in images are associated with high frequency components

High pass filters – only pass the high frequencies, drop the low ones

High pass frequencies are precisely the reverse of low pass filters, so:

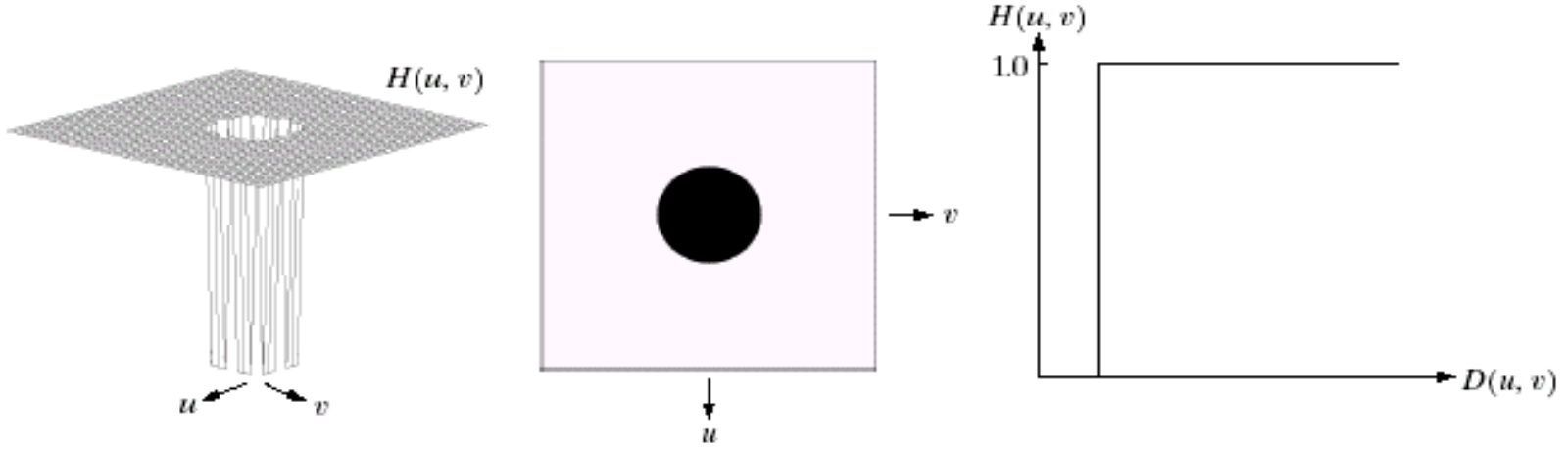
$$H_{hp}(u, v) = 1 - H_{lp}(u, v)$$

Ideal High pass filter

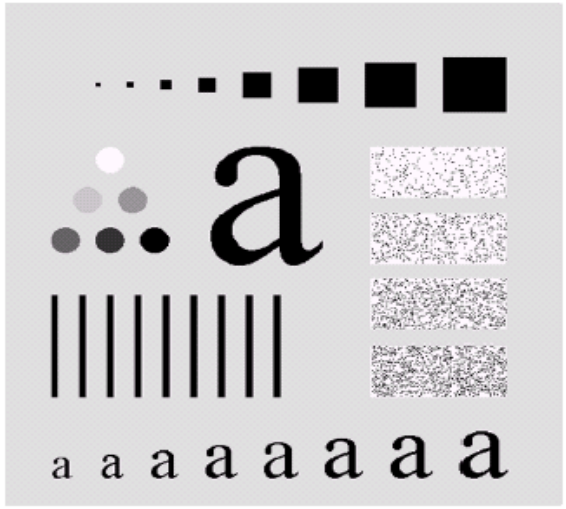
The ideal high pass filter is given as:

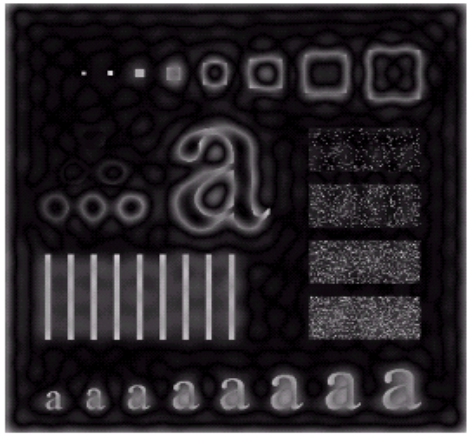
$$H(u,v) = \begin{cases} 0 \text{ if } D(u,v) \le D_0 \\ 1 \text{ if } D(u,v) > D_0 \end{cases}$$

where D_0 is the cut off distance as before

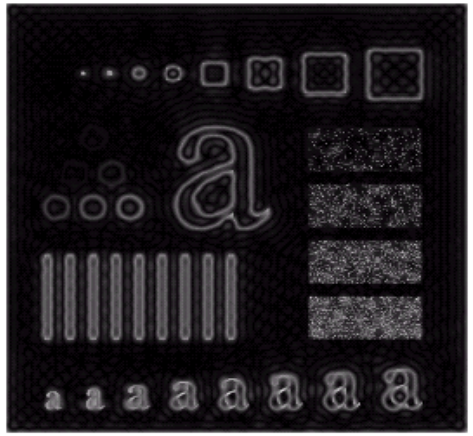


Ideal High pass filter

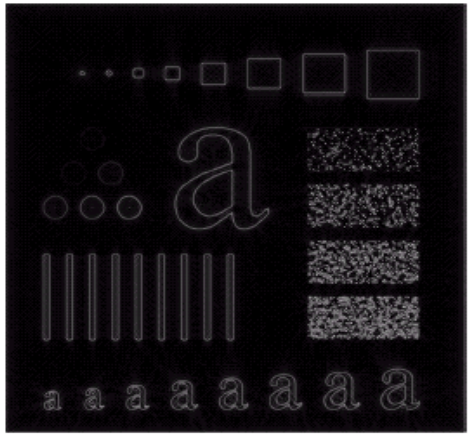




Results of ideal high pass filtering with D_0 = 15



Results of ideal high pass filtering with D_0 = 30

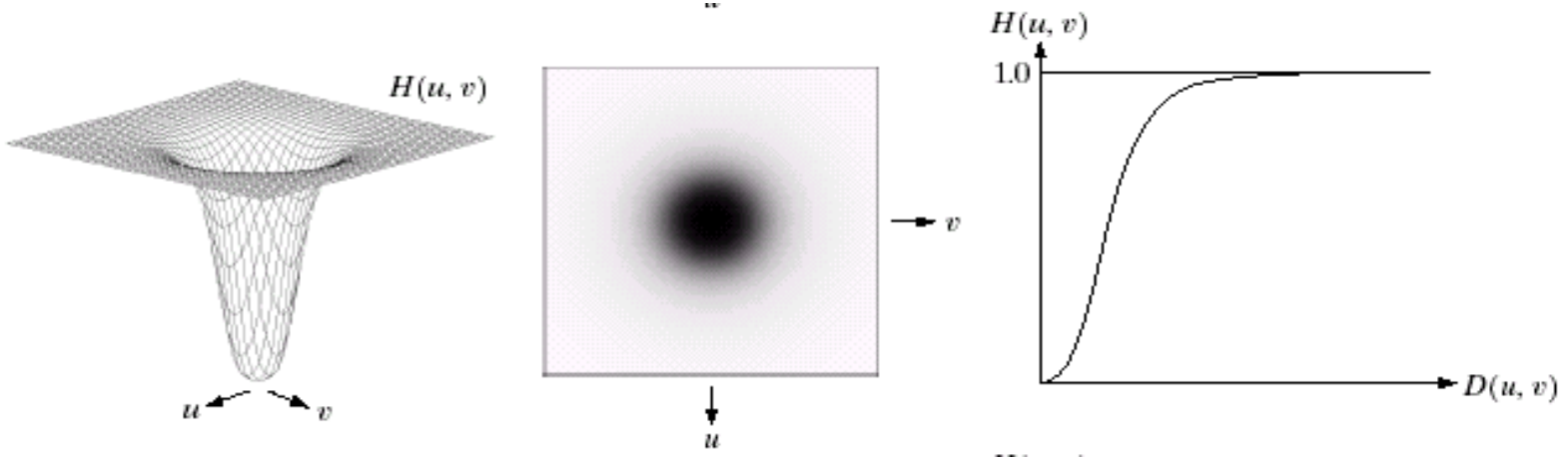


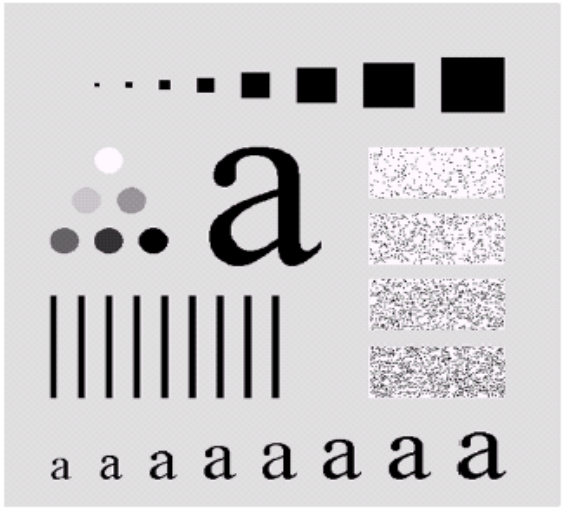
Results of ideal high pass filtering with D_0 = 80

The Butterworth high pass filter is given as:

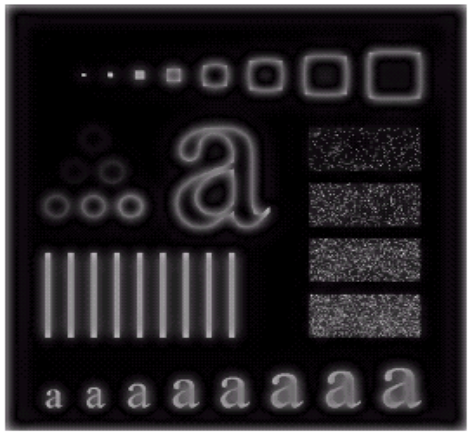
$$H(u,v) = \frac{1}{1+[D_0 / D(u,v)]^{2n}}$$

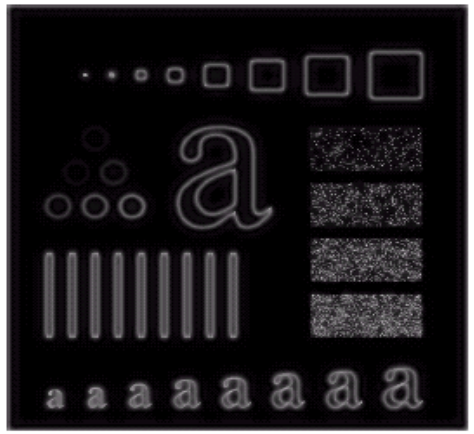
where n is the order and D_0 is the cut off distance as before





Results of Butterworth high pass filtering of order 2 with D_0 = 15





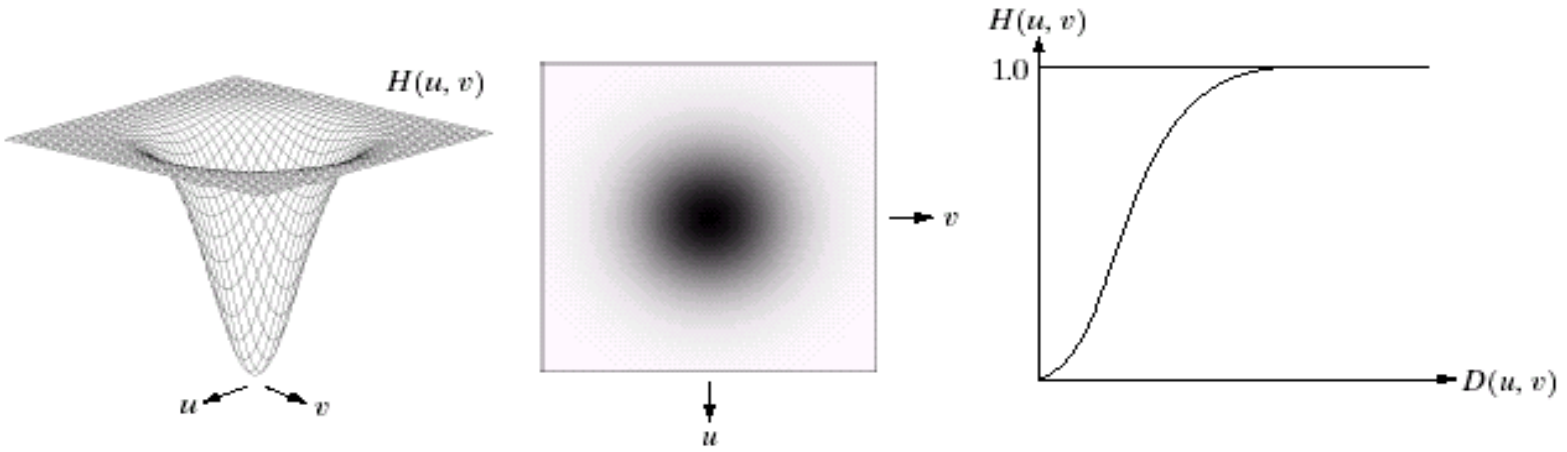
Results of Butterworth high pass filtering of order 2 with D_0 = 80

Results of Butterworth high pass filtering of order 2 with D_0 = 30

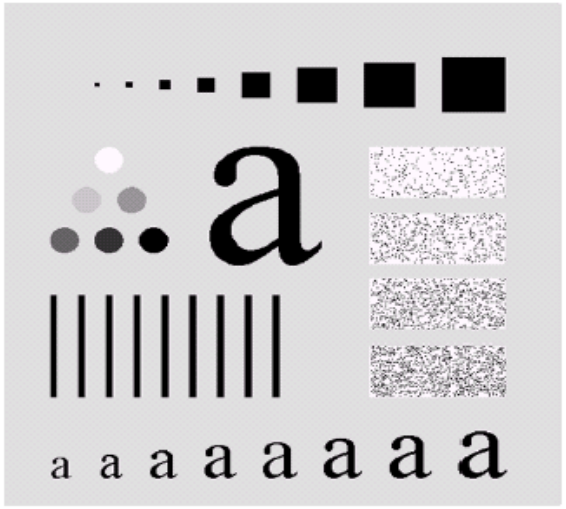
The Gaussian high pass filter is given as:

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0{}^2}$$

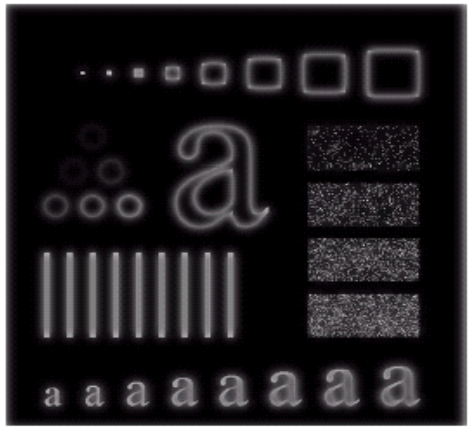
where D_0 is the cut off distance as before

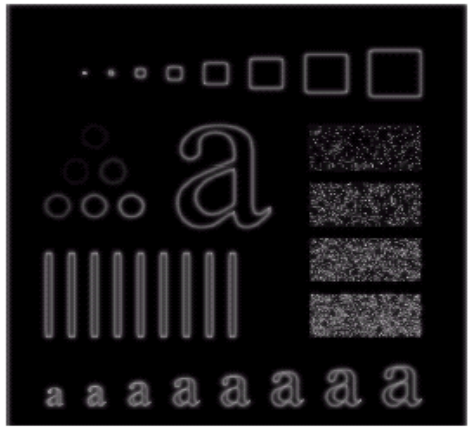


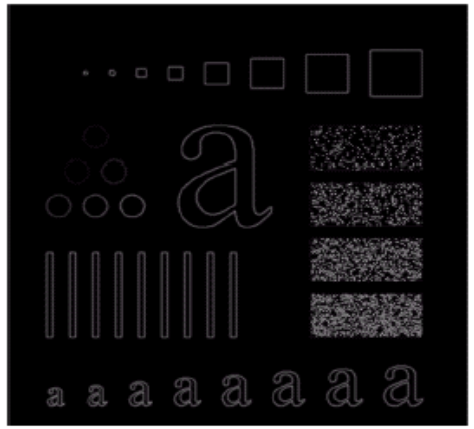
Gaussian High Pass Filter



Results of Gaussian high pass filtering with D_0 = 15



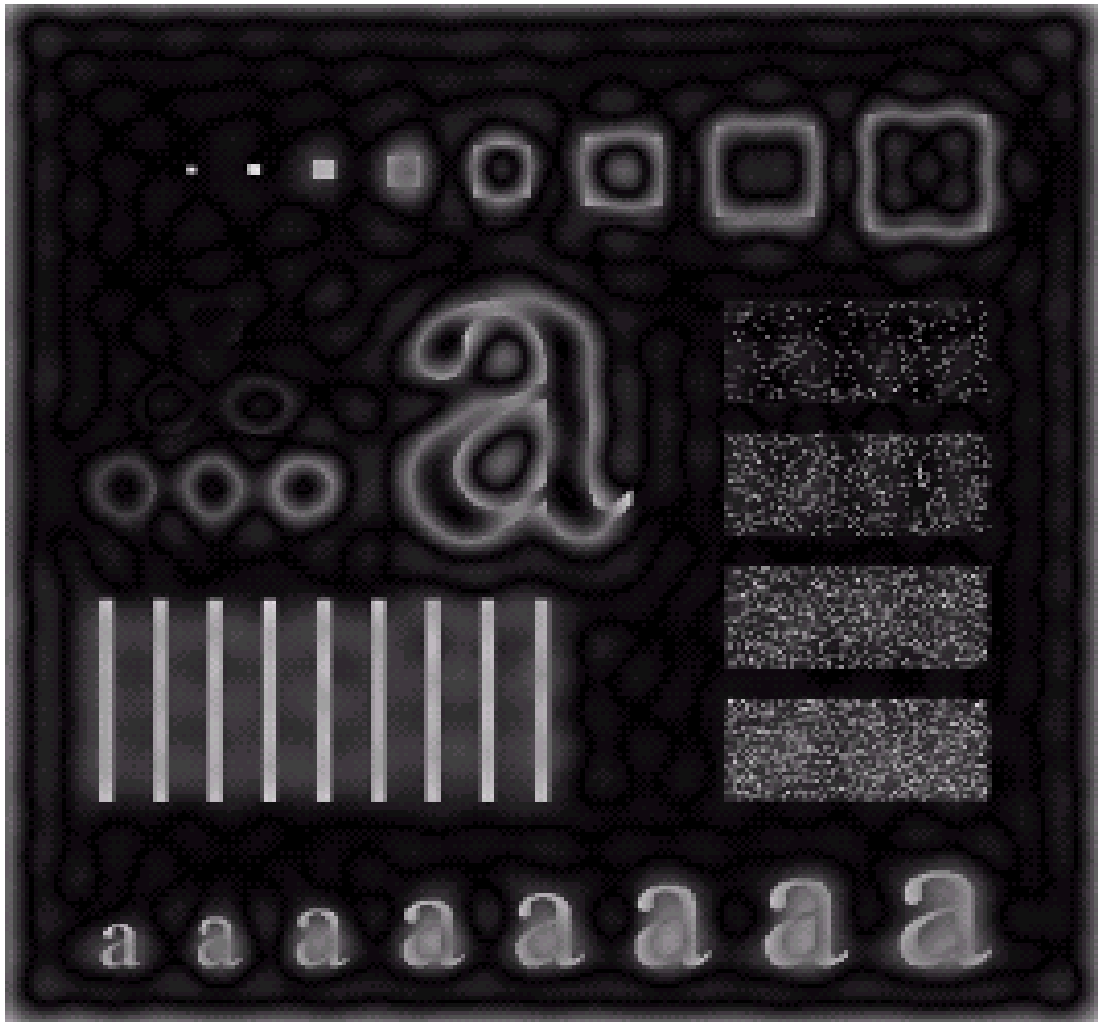




Results of Gaussian high pass filtering with D_0 = 80

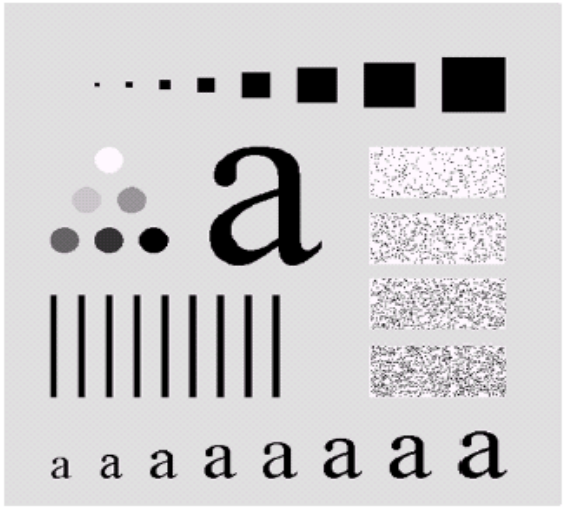
Results of Gaussian high pass filtering with D_0 = 30

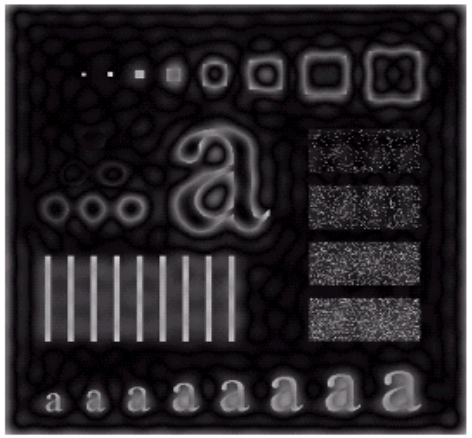
Gaussian High Pass Filter



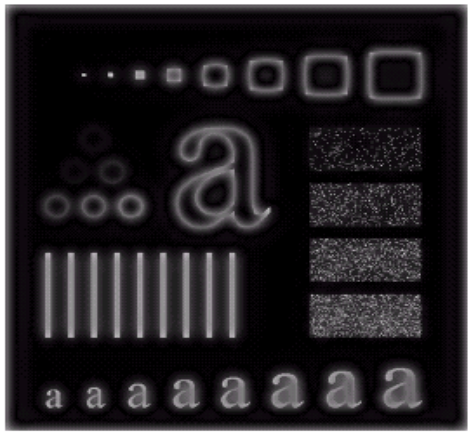
Results of ideal high pass filtering with D_0 = 15

Comparison

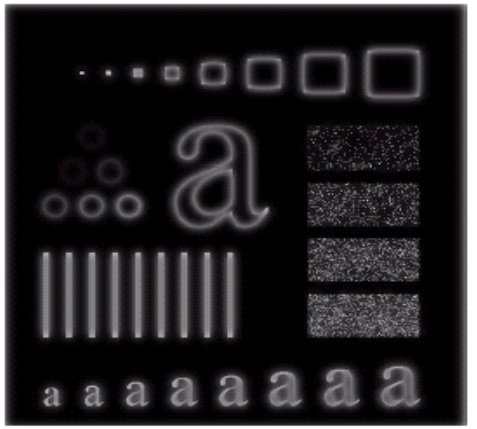




Results of ideal high pass filtering with D_0 = 15



Results of Butterworth high pass filtering of order 2 with D_0 = 15



Results of Gaussian high pass filtering with D_0 = 15

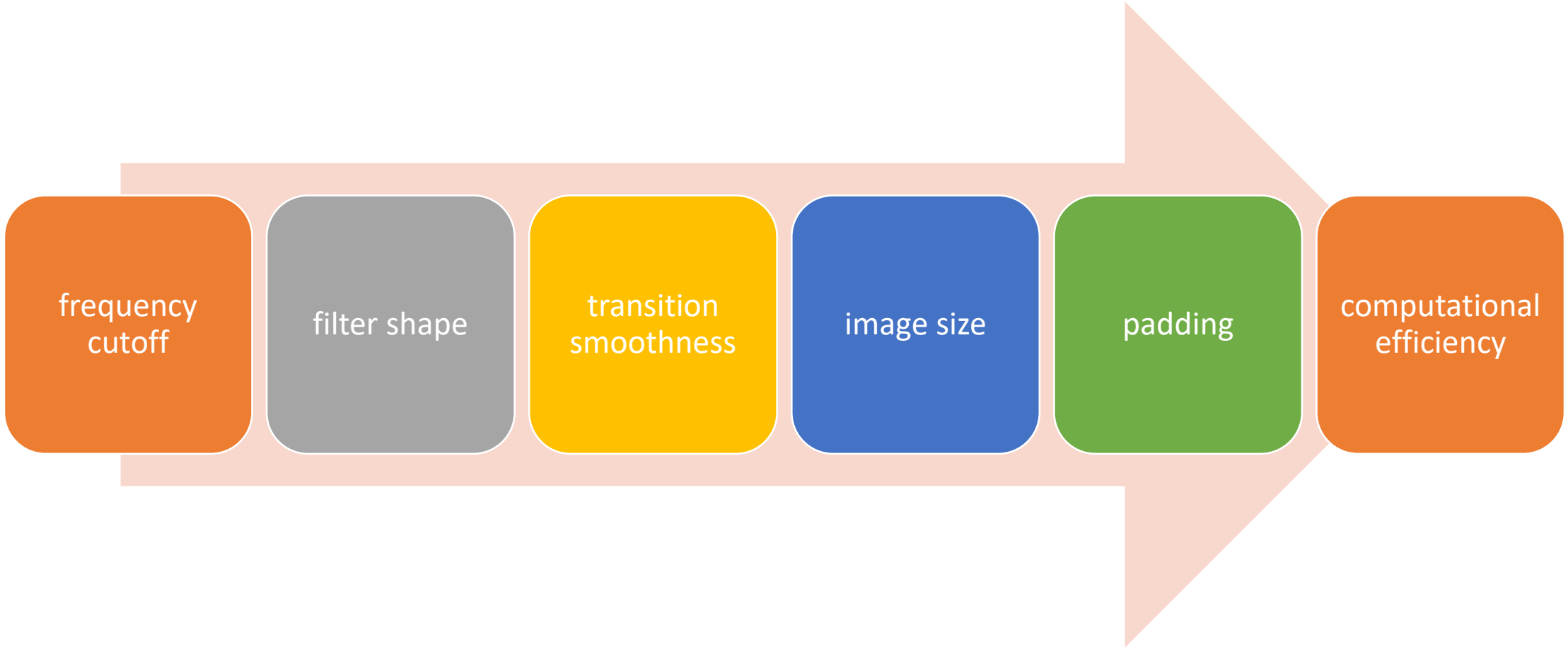
How do Ideal, Butterworth, and Gaussian filters differ in terms of frequency response and real-world applicability?

Ideal filters have abrupt transitions and cause artifacts

Butterworth filters provide smoother transitions

Gaussian filters offer the smoothest response with minimal artifacts.

What practical considerations must be taken when designing a filter in the frequency domain?



How can a Band-Pass Filter be used to isolate specific features in an image?

By allowing a specific frequency band to pass, Band-Pass Filters can isolate patterns or textures corresponding to that frequency range.

What are some real-world applications where frequency domain filtering significantly outperforms spatial filtering?

Applications include medical imaging (e.g., MRI), satellite image enhancement, and periodic noise removal.

How can you visualize and interpret frequency spectra to determine the right filtering approach?

Bright areas in the spectrum indicate strong frequency components; their position and shape guide the selection of appropriate filters.

Assignment

1. Load a grayscale image and apply the following filters in the frequency domain:
    a) Ideal Low-Pass
    b) Gaussian Low-Pass
    c) High-Pass
2. Visualize the frequency spectrum (magnitude) before and after filtering.
3. Compare the results with the same filters applied in the spatial domain.
4. Comment on the differences in edge clarity, noise suppression, and artifacts.